1. INTRODUCTION

In a variety of applications such as spray combustion, spray drying, etc., the evaporation rate is essentially related to the transient liquid heating (or cooling) and internal drop dynamics. This problem is investigated both numerically and experimentally for a single isolated drop suddenly exposed to an unsaturated gaseous environment.

The proposed numerical model includes the effect of the transient cooling in the liquid phase, as well as the effect of variable thermodynamic and transport properties. The results are compared with slowly evaporating ethanol drops of 0.4mm dia., discharged into normal atmosphere.

It was found that calculated evaporation rate is initially underpredicted, approaching however measured contraction rates of the drop after relatively short time (10ms). Observed initial intensification of the evaporation rate is assumed to be related to the large amplitude oscillations of the drop. A simple, semi-empirical modification of the model, which lumps the effects of internal circulation into "effective conductivity", and uses modified correlation coefficients for enhanced thermal and mass transfer in the gas phase is compared with the experimental results.

2. EVAPORATION MODEL

* one-dimensional, spherically symmetric evaporation
* single drop, assumed spherical through its lifetime
* initially uniform temperature distribution in the liquid changes during evaporation
* temperature and pressure of the external gas is modified during evaporation history (finite external gas volume) or assumed to be constant (infinite environment).
* the gas properties are calculated using so called "1/3 rule" for temperature

The external heat flow rate dQ_e/dt absorbed through the drop surface is equal to the rate of change of drop **internal energy** dQ_i/dt plus the rate of **energy used for evaporation** of an incremental volume of the liquid dQ_l/dt , i.e.:

$$dQ_e/dt + dQ_i/dt + dQ_l/dt = dQsum(T_s)/dt = 0.$$

T_s is the temperature at the drop surface.

The history of the evaporating drop is calculated iteratively, step-by-step, modifying drop size $R(t)$, internal temperature distribution $T(r,t)$, relative drop velocity $V(t)$ and thermodynamic properties of both phases.

2.1 Modelling of the heat and mass transfer

The external heat transfer is given by:

$$Q_e(t) = 2\pi R(t)\lambda(t)(T_\infty - Ts(t))Nu_H(t)$$

λ is heat conduction coefficient, Nu_H is the Nusselt number modified for relative drop-gas movement:

$$Nu_H = 2(1 + 0.3Re^{1/2}Pr^{1/3}) \quad (Ranz\ \&\ Marshall).$$

Pr and Re are the Prandtl and Reynolds numbers for the drop.

Heat transfer to the drop interior

$$dQ_i/dt = 4\pi R(t)^2\lambda(t)dT(r,t)/dr|_{r=R(t)}$$

is calculated using analytical solutions for the flow of heat in a solid sphere with a variable temperature distribution (Carslaw & Jaeger). Hence, the surface temperature $T(r = R_i, t_i)$ and the drop radius are functions of time and they are recalculated for each time step t_i, using $T(r, t_{i-1})$ as the initial temperature function. This is the most time consuming step of the presented scheme.

The evaporation rate is calculated with help of a multirange model of Gyramathy (1982), which in the present case reduces to Maxwell-Stephan model for diffusive evaporation :

$$dm/dt = \frac{2\pi R(t)D\rho_\infty Rm(\overline{P}v_\infty - \overline{P}s(t))Nu_M(t)}{Rv(1 - \overline{P}v_\infty)}$$

Dashed symbols indicate normalization with the ambient pressure P_∞, D is diffusion constant, Pv_∞ - vapour pressure in the infinity and ρ_∞ - gas density in the infinity. Rv and Rm - are vapour and mean gas constants.

The modified Nusselt number for the mass transfer (Sherwood number) depends on the Schmidt number Sc, Reynolds number and normalized gas pressure Pg_∞:

$$Nu_M = 2(1 + 0.3Re^{\frac{1}{2}}Sc^{\frac{1}{3}}\overline{P}g_\infty^{\frac{1}{3}})$$

2.2 Effect of drop oscillations

Following modifications of the model are applied:

Internal circulation $\Longrightarrow$ "Effective Conductivity Model":

$\lambda_{osc} = \lambda(1 + \chi(Re_l Pr_l)^{\frac{1}{2}})$, where χ is found to be 0.08.

Enhanced heat and mass transfer to the gas $\Longrightarrow$ Modified correlations coefficients for Nusselt and Sherwood numbers:

$Nu_{osc} = Nu(1 + \zeta Re_g^{0.7}Pr^{\frac{1}{3}})$, where ζ is found to be 12.5.

Re_l, Re_g are Reynolds numbers for the internal and external flow, generated by drop oscillations.

3. Calculated Example

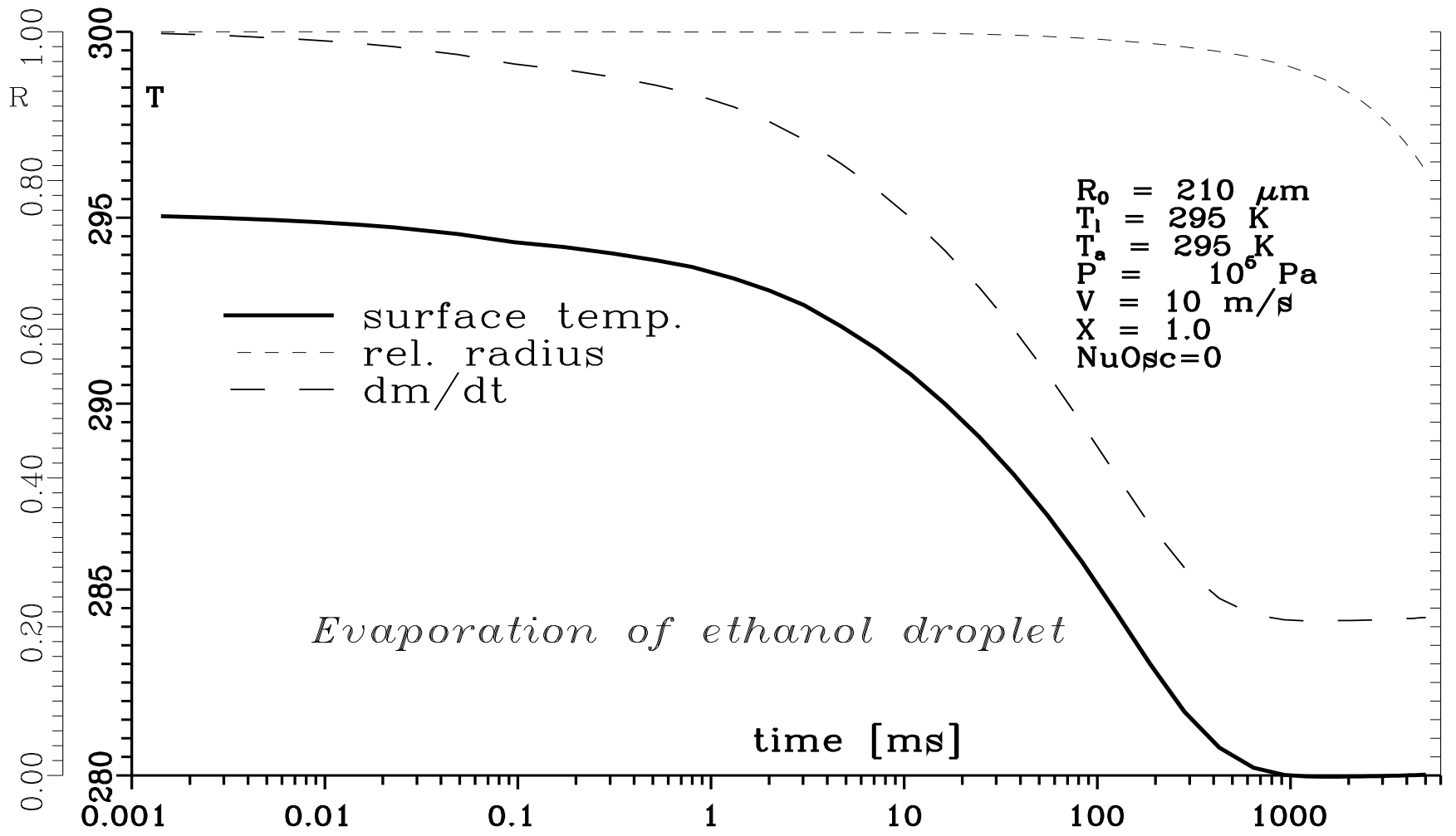


Figure 1 shows an example of calculated history of ethanol drop evaporating under normal atmospheric pressure. The initial drop radius is 210 μm and initial temperatures of the drop and surrounding neutral gas are equal 295 K. The drop initial velocity is 10 m/s. The calculated variation of the drop diameter with time is shown together with its surface temperature and evaporation mass rate.

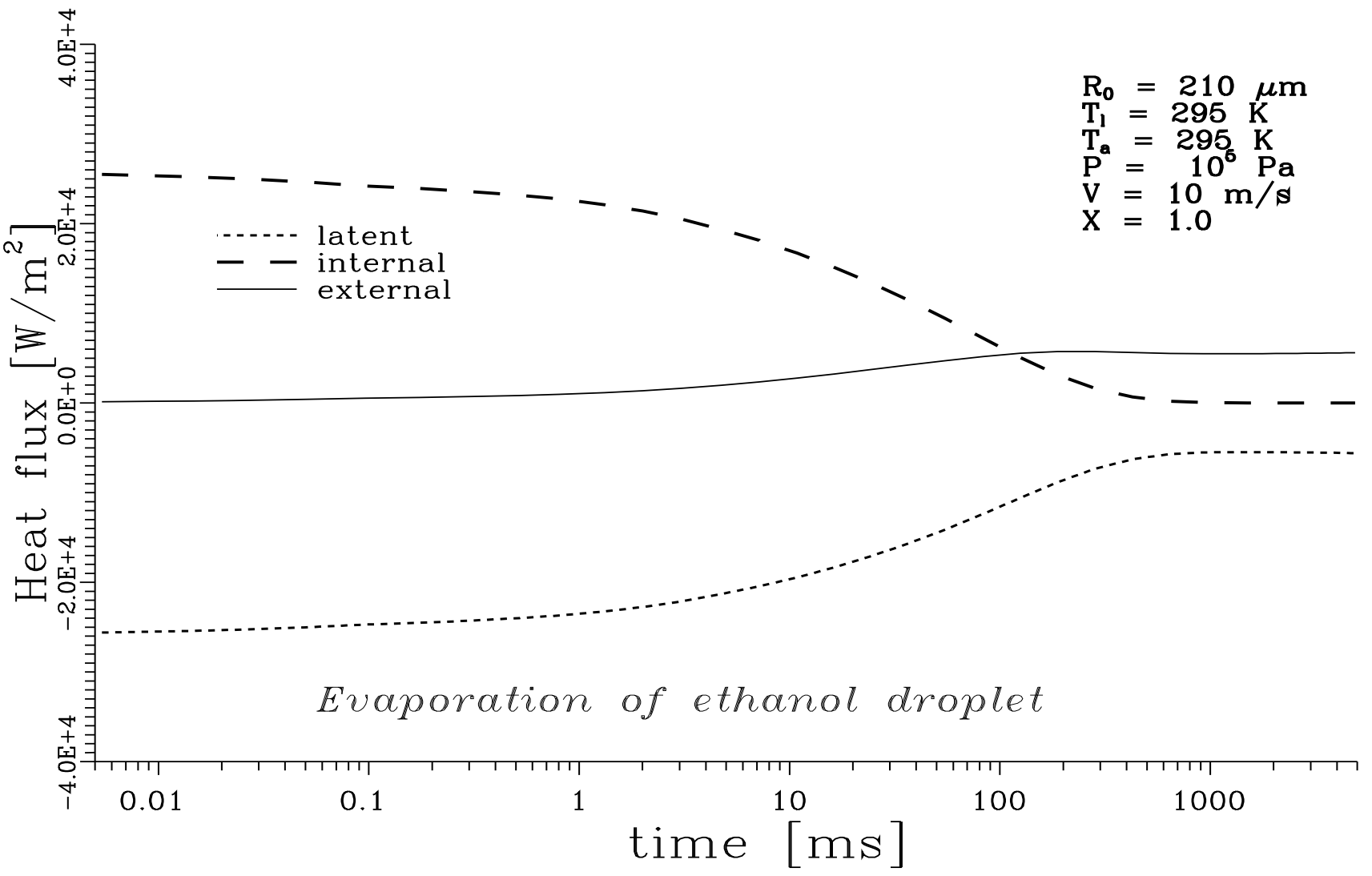


Fig. 2. *Temporal variation of the internal, external and latent heat fluxes for the evaporating drop from figure 1.*

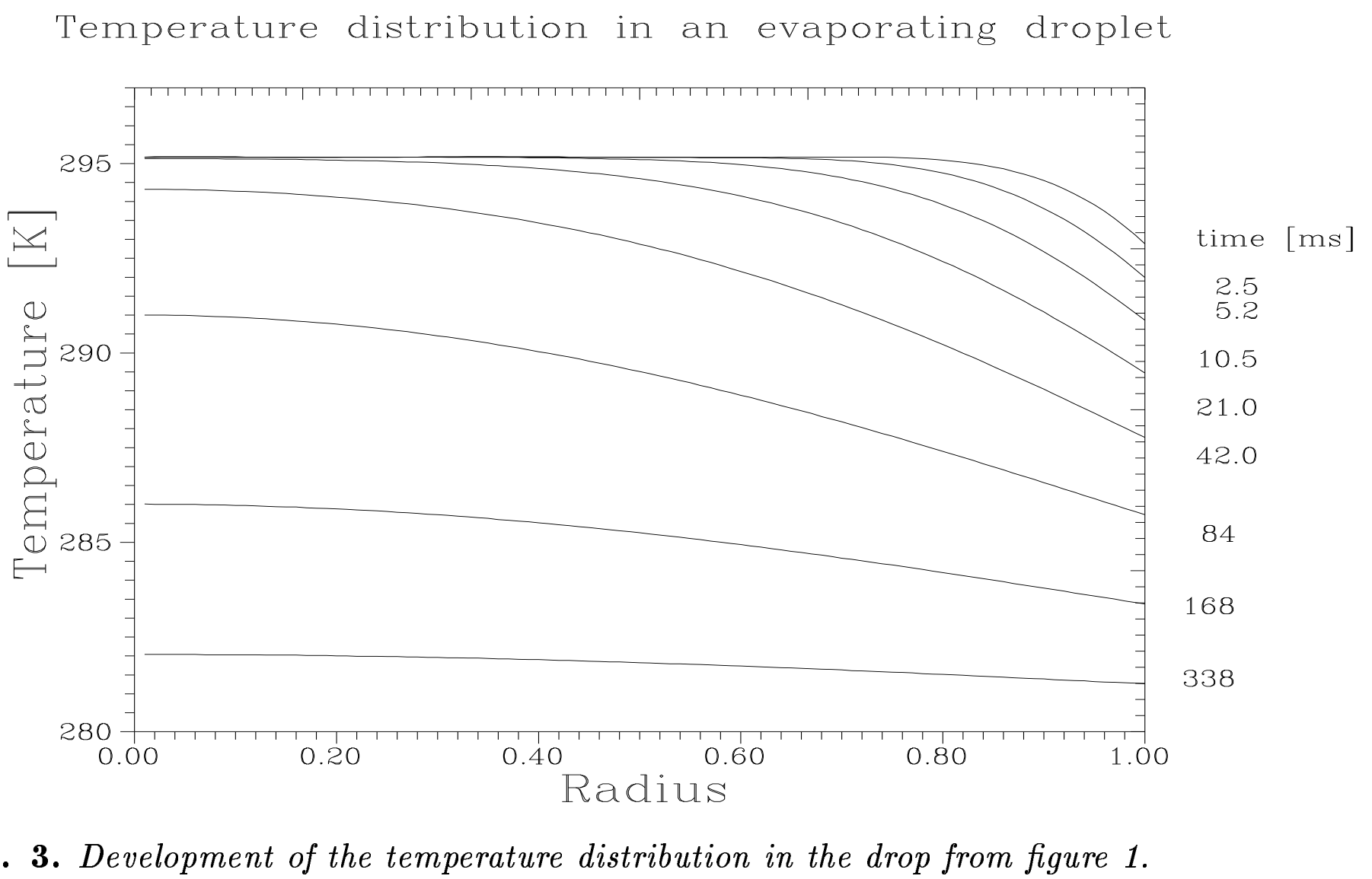
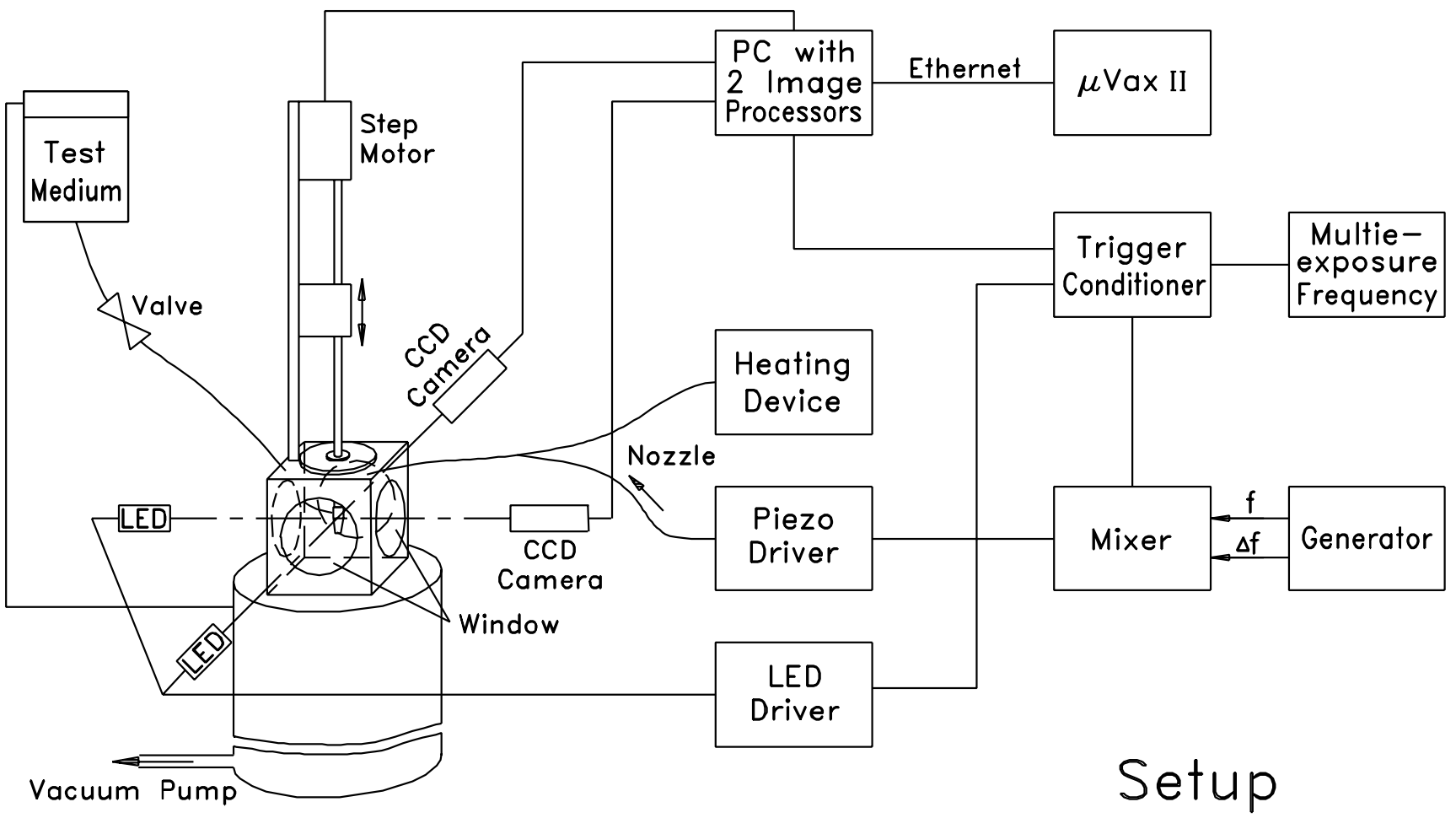


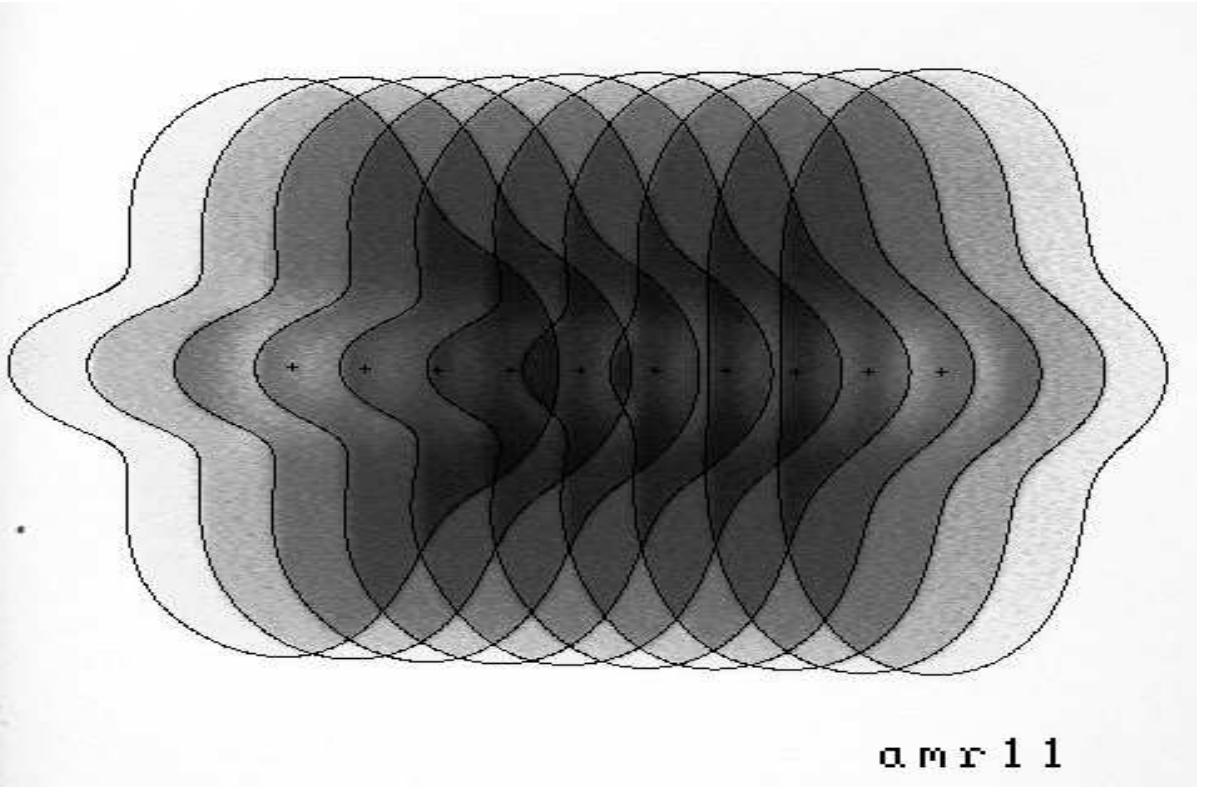
Fig. 3. *Development of the temperature distribution in the drop from figure 1.*

4. EXPERIMENT



The initial behaviour of slowly evaporating ethanol drop was analyzed during the first 15ms and compared with the proposed numerical modeling. drops at room temperature (295K) are dispersed in air of normal atmospheric pressure and observed by two CCD cameras from two perpendicular directions in bright field illumination. Digitized images of drops are used to determined their radius and oscillation frequency. The measured time variation of the drop diameter gives directly the drop evaporation rate. An additional analysis of the drop oscillation frequency is used to measure time dependent variation of the surface tension of the drop. The variation of the surface tension with temperature offers an unique possibility of measuring the surface temperature of an evaporating drop.

Fig.4. *Image and evaluated contour (solid line) of a multi-exposed oscillating ethanol droplet. Equilibrium radius 0.210mm. Strobe frequency 128kHz, 10 flashes of 200ns duration. Evaporation rate is obtained from the temporal variation of the evaluated volume; analysis of the oscillation frequency is used to measure the time dependent variation of the surface tension*



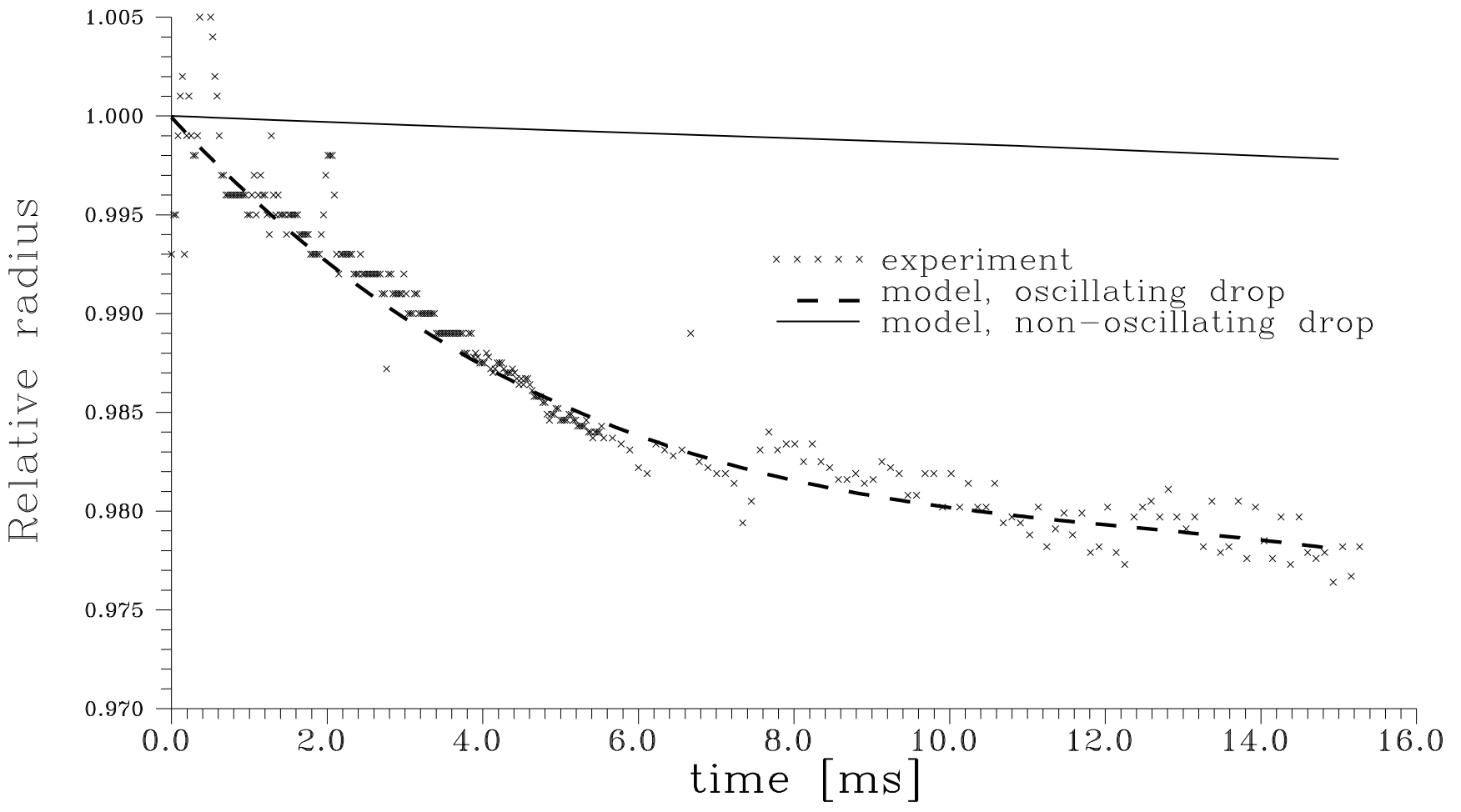


Figure 5. shows variation of the drop equilibrium radius measured from 270 drop images. The observed decrease of the drop radius is very small (weak evaporation). The observed initial evaporation rate are underpredicted by "non-oscillating drop model". After about 10ms the measured contraction rate of the drop radius decreases to the calculated values, which are of order $10^{-4} m/s$.
The observed initial intensification of the evaporation rate can be related to the large oscillation amplitudes of the drops (figure 6). The relative motion of the drop surface induces internal and external circulation of the fluid. It modifies the heat transfer in the drop and also the heat and mass transfer between drop and gas. The second curve, "oscillating drop model", fits quite well to the experimental data.

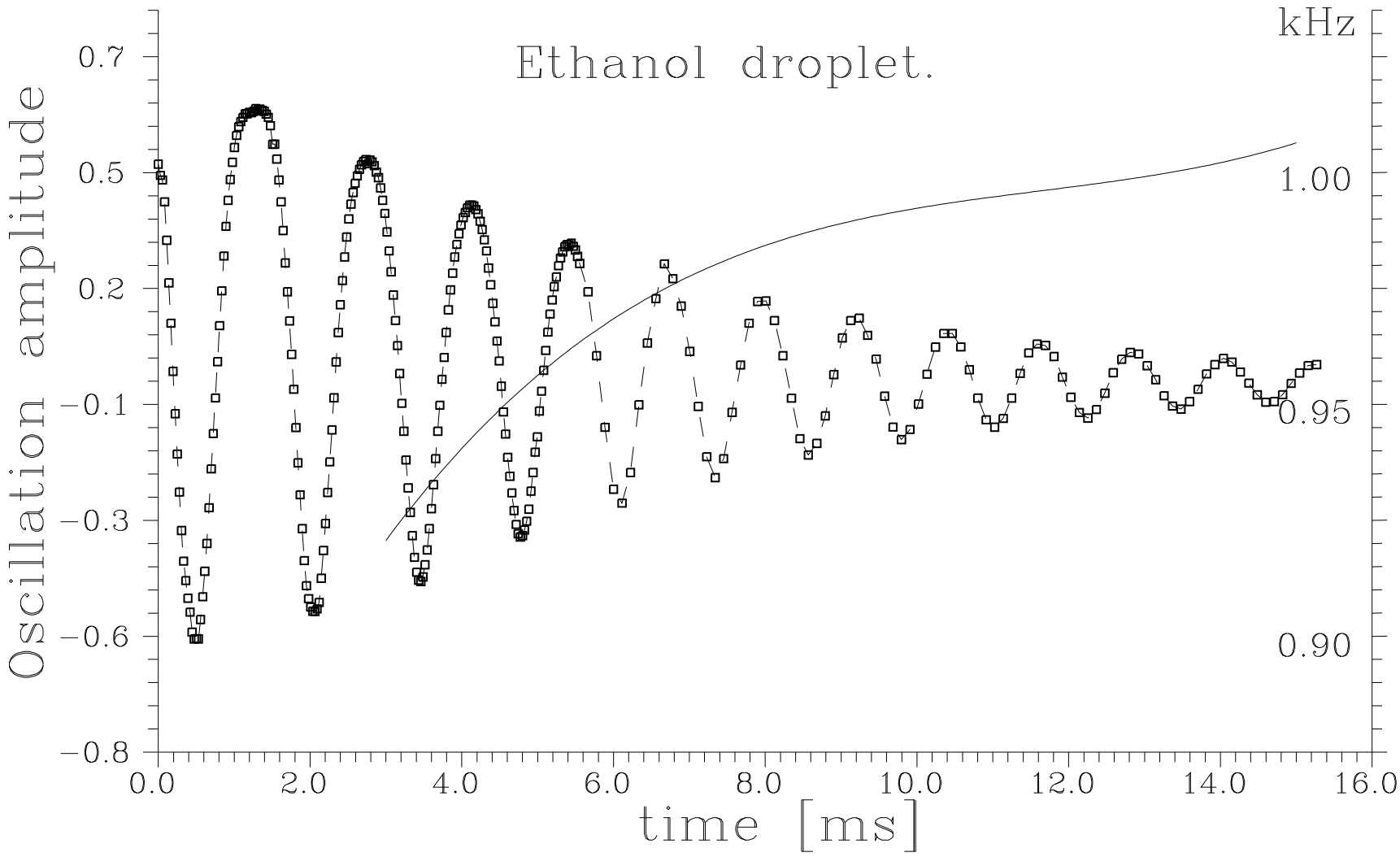


Fig.6. *Measured amplitude of the fundamental oscillation mode for the evaporating ethanol drop. Solid line - approximate variation with time of the oscillation frequency.*

The measured frequency of the drop oscillation increases during analyzed time. It suggests an increment of the surface tension, meaning cooling of the drop surface. Detailed analysis of the drop oscillation needs to take into account also nonlinear effects, present at large amplitudes. Therefore only part of the measured oscillation frequency shift is due to the variable surface temperature (i.e. surface tension) of ethanol.

5. FINAL REMARKS

Even though the initial stage of evaporation, which is controlled by the drop thermal relaxation and oscillations, amounts to only 10-20% of its total life time, the importance of this transients is of prime interest in many practical applications (e.g. of fuel injection in automotive engines, where the whole evaporation time is determined by the engine cycle).

The present simple numerical model, which takes into account transient heating of the drop, can properly describe its initial history (decrease of radius and surface temperature). It seems that its additional modifications permit to predict also the influence of the drop oscillations on the evaporation rate.
Further application of a newly developed nonlinear theory of drop oscillation will allow us to verify the calculated surface temperature of evaporating drops.