AN EXPERIMENTAL BENCHMARK FOR FREEZING WATER IN THE CUBIC CAVITY

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ABSTRACT. The problem of transient natural convection of freezing water in a cube-shaped cavity is investigated experimentally. By immersing the cavity in the external water bath, the effect of the heat transfer through the side walls is studied. The velocity and temperature fields are measured using liquid crystal tracers. The transient development of the ice/water interface is measured.

INTRODUCTION

In number of manufacturing processes a solid material is formed by the freezing of a liquid. Due to the problem complexity application of numerical methods to these engineering problems is not a trivial task, and the experimental verification gains its importance. To avoid geometrical complications and uncertainty of the thermophysical properties, often a simple model of water freezing in a differentially heated cavity is used for the code testing purposes[1]. However, the available comparisons with experiment are very limited. This is perhaps, due to the fact that most of the accessible experimental data on freezing are limited to the general observations of the phase change front and point measurements of the flow velocity and temperature. The aim of the present investigation is to create an experimental benchmark for the problem of natural convection in freezing water. It consists of the transient data on the interface position, and the temperature and velocity fields - gained by the new full field experimental methods. The experimental results are compared with the numerical simulations performed using the modified 3-D finite difference code FREEZE3D[2]. The well known phenomena of the highly non-linear relationship between density and temperature in the vicinity of the freezing point puts an additional challenging complexion on the numerical simulation of the problem.

FORMULATION OF THE PROBLEM

We consider the convective flow in a box filled with a viscous heat conducting liquid, which in this case is distillated water. The fluid density, viscosity, thermal conductivity and heat capacity are assumed to be temperature dependent. The flow takes place in a container with an aspect ratio of one, its two opposite vertical walls are assumed isothermal. One of the vertical walls is held at temperature T_c= -10°C. It is below the freezing temperature of the liquid T_f= 0°C, hence the solid forms there. The opposite vertical wall is held at temperature T_h = 10°C. The other four walls of low thermal conductivity allow the entry of heat from the external fluid which surrounds the cavity. Two cases are considered. In the first one the cavity is surrounded by a laminar air stream at temperature T_{ext}=25°C, in the second case the water bath of constant temperature T_{ext} = T_h entraps the cavity. The temperature field at the inner surfaces of the walls adjusts itself depending on both the flow inside the box and the heat flux through and along the walls. Two initial conditions are investigated. In the first case, we call it here the „cold start”, the initial fluid temperature and temperature of all six walls equals T_o = 0.5°C, i.e. it is just above the freezing point. The null initial velocity flow field is assumed. The freezing experiment starts, when both hot wall and cold wall temperatures are suddenly changed to +10°C and -10°C, respectively. In the second initial condition, we call it the „warm start”, the freezing starts after the steady convection pattern is established in the cavity. This initial flow state corresponds to natural convection without phase change in the differentially heated cavity, with the temperature of the cold wall set to T_c=0°C. The freezing experiment starts, when at time t=0, the cold wall temperature suddenly drops to T_c= -10°C.

The three basic dimensionless parameters defining the problem the Rayleigh number (Ra) the Prandtl number (Pr) and the Stefan number (Ste) are given as

$$\mathrm{Ra} = \frac{g\beta\,\Delta T H^3}{\alpha\nu}\ ,\ \mathrm{Pr} = \frac{\nu}{\alpha}\ \text{ and }\ \mathrm{Ste} = \frac{c_p\,\Delta T}{L_f},$$

where $\Delta T = T_h - T_r$ is the temperature difference of the hot wall T_h and the phase interface T_r. In the above definitions g, H, α, β, ν, c_p, L_f, denote respectively the gravitational acceleration, the cavity height, the thermal diffusivity, the coefficient of thermal expansion, the kinematic viscosity, the specific heat and the latent heat of fusion. The physical properties of fluid used above are taken for the arbitrary selected reference temperature T_r= 0°C. The density function was obtained by fitting the fourth order polynomial to the data collected by Kohlrausch[3] (fit error 0.02%):

$$\rho = 999.840281167108 + 0.0673268037314653 \cdot t - 0.00894484552601798 \cdot t^2 + \\ + 8.78462866500416 \cdot 10^{-5} \cdot t^3 - 6.62139792627547 \cdot 10^{-7} \cdot t^4,$$

where the temperature t is given in degrees Celsius.

The remaining relationships for the water heat capacity, the thermal conductivity and the viscosity are those given by Reizes et al.[4] The thermophysical properties of ice were assumed to be constant and equal: 916.8kg/m^3 for the density, 2.26W/mK for the thermal conductivity, and 2.116kJ/kgK for the heat capacity. The latent heat equals L_f = 335kJ/kg. The thermal conductivity, heat capacity and density of the Plexiglas used were measured. When solving the energy equation for the side walls, the values of 0.195W/mK for the thermal conductivity, and $1.19 \cdot 10^{-7}$m^2/s for the thermal diffusivity were used. The heat transfer coefficient h used for modelling the convective heat flux from the external fluid was taken to be equal 1000W/Km^2 for the forced convection of water and 20W/Km^2 for the air flow.

NUMERICAL

A numerical simulation of the problem was performed using a finite difference model of the Navier-Stokes and energy equations. The modified version of three-dimensional numerical code FREEZE3D[2] have been used. The code allows to study transient convection with phase change in a fluid with temperature-dependent properties. The vorticity-vector potential formulation[5] is used in the code. As the physical domain changes in shape, the interface boundary grid is generated at each time step. When simulating experimental conditions, the main problem which arises is the proper definition of thermal boundary conditions (TBC). In our study three different approaches were used to test their effect on the final solution. Specifically, the TBC for the non-isothermal walls have been either computed using the 1-D or 3-D modelling of the heat flux through/along these walls or the idealized adiabatic walls were assumed. In the 1-D TBC approach, so called „convective" TBC were imposed, using the general form of non-dimensional conditions for the temperature calculated from physical properties of fluid/wall material and arbitrary specified heat transfer coefficient. In the 3-D TBC case the additional energy equation for the physical walls has been incorporated into the numerical model, and the coupled fluid-solid heat conduction problem was solved. Solutions were obtained using a 21^3 mesh points for the fluid and additional $21x21x10$ mesh point for the solidus. Using 3-D TBC 5 additional grid points were located in each of the four side walls. To test the mesh dependence selected cases were calculated increasing number of grid points into 31^3 for the fluid domain. To start the freezing calculations the initial grid for solid phase must exist. Hence, it was assumed that at the first instance the cold wall is already covered with ice layer of non-dimensional height equal 0.02.

Several cases, covering both the preliminary study of the effect of density anomaly on the fluid flow, as well several different approaches to modelling experimental conditions, were simulated numerically. The complete collection of the data obtained will be given elsewhere[8]. To select the most appropriate experimental conditions the preliminary numerical study was performed for the hot wall temperature set to 5°C, 7.5°C, 10°C, 12.5°C and 15°C. In all cases the cold wall temperature was set to -10°C. In this temperature range large variation of the flow pattern and freezing rates were observed, from single counter-clockwise „anomalous" circulation at lower temperature range, to the similar clockwise „vortex" for the highest temperature (15°C). In the middle range, setting 10°C at the hot wall, well defined thermal stratification was present. Two main vortices running in the opposite directions were separated by the region of density extremum. High velocity and temperature gradients in the region of two colliding flow patterns create challenging conditions for the numerical modelling. Hence, this flow condition was selected for the experimental investigations.

EXPERIMENTAL

The experimental set-up used to acquire temperature and velocity fields consists of the convection box, a halogen tube lamp, the 3CCD colour camera (KYF55 JVC) and the 32-bit frame grabber (IC-PCI ITI). The 24-bit colour images of 560x560 pixels were acquired using 64MB Pentium 133 computer. The convection box, of 38mm inner dimension, have two isothermal walls made of a black anodised metal. The four non-isothermal walls were made of 6 mm (experiments with air as external fluid) and 9mm thick Plexiglas (external water bath). The isothermal walls were maintained at a constant temperature by anti-freeze coolant flowing through the attached antechamber. The temperature of the cooling and heating liquids and that of water in the bath surrounding the four non-adiabatic walls was controlled by thermostats. The experiment started by opening abruptly the inlet valves to the coolant passages. The temperature of cold and hot wall was -10°C and +10°C, respectively. Distilled water was selected as a flow medium for its well known thermophysical properties and well defined temperature of the phase change. The flow field was illuminated with a 2mm thin sheet of white light from a specially constructed halogen lamp, and observed at the perpendicular direction.

Table 1. Experimental runs.

Case	Initial start	Run	Experiment	Tracers
External flow: air 6mm Plexiglas side walls	„cold"	#1	PIV	Pine
	„cold"	#2	PIV	Pine
	„cold"	#3	PIV + PIT	TLC – B
	„warm"	#4	PIV + PIT	TLC – D
External flow: water 9 mm Plexiglas side walls	„warm"	#5	PIV + PIT	TLC – C
	„warm"	#6	PIV + PIT	TLC – B
	„warm"	#7	PIV + PIT	TLC – D
	„warm"	#8	PIV	Lycopodium
	„warm"	#9	Front location	–

Both velocity and temperature fields were monitored using uncoated Thermochromic Liquid Crystal (TLC) tracers[6]. The mean diameter of the TLC tracers was about 50μm. Particle Image Thermometry (PIT), which is based on temperature-dependent reflectivity of TLCs, was applied to measure 2-D temperature fields. If the liquid crystals are illuminated with white light, then the colour of the light they reflect changes from red to blue when the temperature is raised. This occurs within a well defined temperature range (the so-called colour play range), which depends on the type of TLCs used. The colour-temperature relationship is strongly non-linear. Hence, the accuracy of the measured temperature depends on the colour (hue) value, and varies from 3% to 10% of the full colour play range. For the TLCs used (TM from Merck) the most sensitive region the colour transition is from red to green and takes place for the temperature variation less then one degree

Celsius. To improve the accuracy of temperature measurements experiments were repeated using four different types of TLCs, so that their combined colour play sensitivity range covered temperatures from -5°C to 14°C.

The 2-D velocity vector distribution was measured by particle image velocimetry (PIV). By this method, the motion of the scattering particles observed in the plane of the illuminating light sheet can be analyzed. For this purpose the colour images of TLC tracers were transformed to B&W intensity images. After applying special filtering techniques bright images of the tracers, well suited for PIV, were obtained. Also some additional experiments were performed using colourless „classical" PIV tracers (*pine pollen* and *lycopodium*). The magnitude and direction of the velocity vectors are determined using a cross-correlation analysis between small sections (interrogation windows) of one pair of images taken at the given time interval. The average particle displacement during given time interval determines the velocity vector representing analyzed section. By moving step by step interrogation widow across the image about 1000 vectors per one pair of images were obtained. The spatial resolution of the method is limited by minimum amount of tracers to be present in the interrogation window. In practice, the minimum window size was 32x32 pixels. On the other hand, dimension of the interrogation window limits maximum detectable displacement. Hence, to improve the accuracy and dynamic of the velocity measurements short sequences of images were taken. The cross-correlation analysis performed between different images of the sequence (time interval between pairs changes), allowed to preserve similar accuracy for both the low and high velocity flow regions. The modified multi-quadratic approach[7] was used to interpolate spurious vectors. To obtain a general view of the flow pattern, several images recorded periodically within a given time interval were added in the computer memory. Displayed images are similar to the multiexposed photographs, showing the flow direction and its structure. This type of visualization is very effective in detecting small re-circulation regions, usually difficult to identify in the velocity field. In all analyzed cases the volume concentration of tracers was very low (below 0.1%), so their effect on the flow and the physical properties of water was negligible small.

Our main interest was directed to collect quantitative information about the phase front position, the velocity and temperature fields for the centre vertical plane of the cavity. The flow images were collected periodically every 60s or 120s, approximately during two hours after cooling was started. The typical set of the experimental data consists of 50-60 sequences of four 24-bit images, and uses approximately 250MB disk space.

The main futures of the experiments performed are collected in Table 1. Two separate experimental set-ups were employed. In the first configuration the cavity was surrounded by air. The heat transfer from the gas environment through the Plexiglas walls is relatively low and its effect on the internal flow was considered to be small. The constant flux of air generated by a low speed fan directed at the cavity was used to make the heat transfer possible uniform. Both the „cold start" and „warm start" experiments were performed for the cavity surrounded by air. To obtain well defined thermal boundary conditions for the side walls the second configuration was selected. Here, the cavity was immersed in the water bath of constant temperature (forced convection). Only „warm start" initial conditions were possible for that case.

SELECTED RESULTS

Large amount of the experimental data were collected. Different experimental runs performed for the same conditions confirmed reproducibility of the experimental technique. The measured interface profiles and velocity fields from the different runs could be matched within 5-8% error. Taking into account the page limit constraints of the conference paper, in the following we have decided to select only the small sample of our results. The detailed collection of the data will be given elsewhere[8].

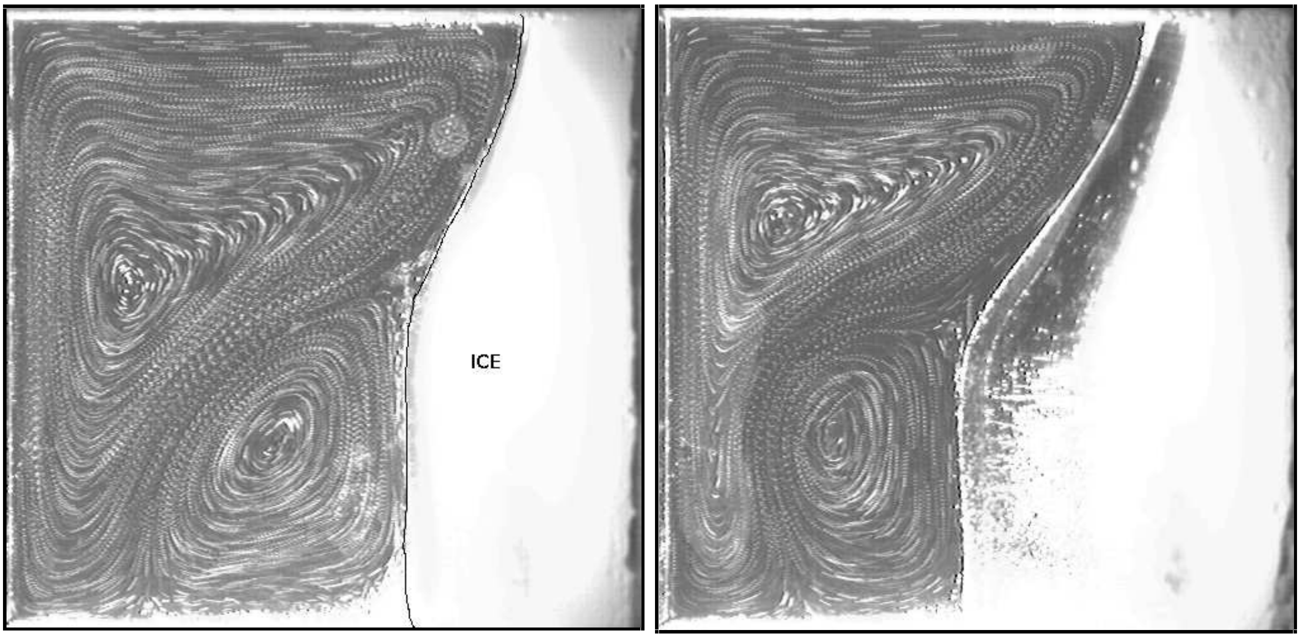

Figure 1. Ice fronts observed for the run #1 at 2340s (right) and 6000s (left) after cooling starts. Superposition of 10 images taken every 0.4s.

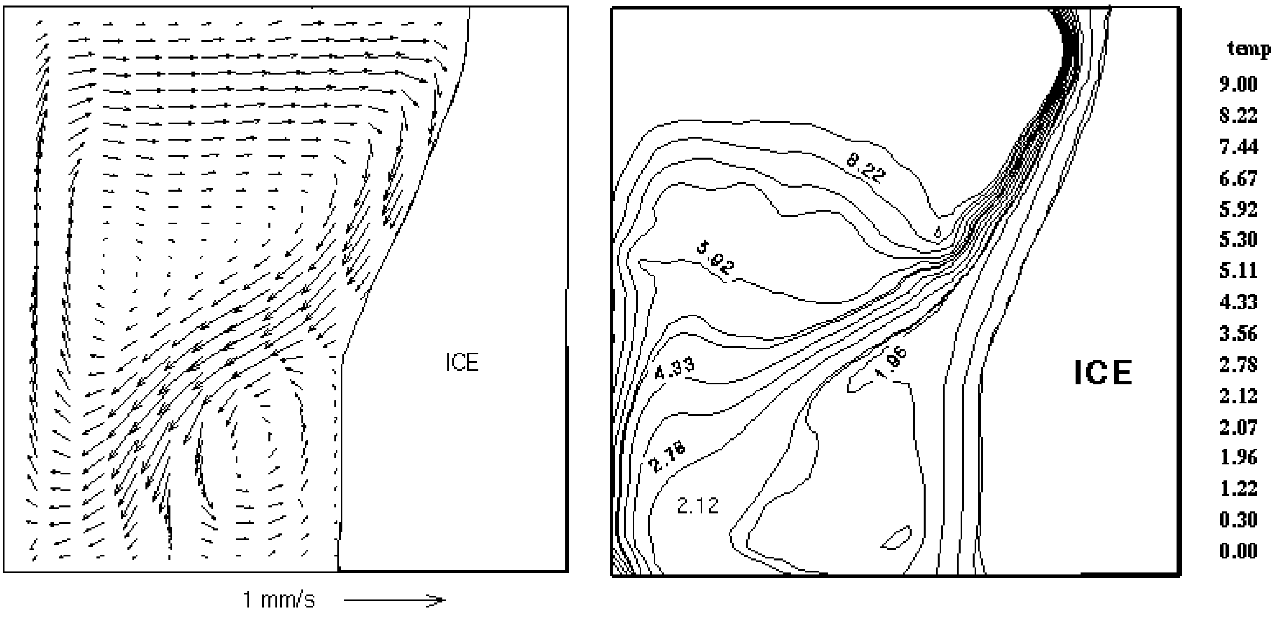

Figure 2. Measured velocity and temperature fields at 3000s; run #4: „warm start", external air flow.

The experiments performed shown that the selected case of phase change problem is far from simplicity and surely worth additional detailed studies. The competing effects of positive and negative buoyancy force, interacting layers of hot and cold liquid create interesting and difficult for modelling flow pattern (Figure 1). Two main circulations, found in the numerical model, are well visible: the upper clockwise circulation transporting the hot liquid to the top wall and back along isotherm of the density extremum, and the lower counter-clockwise circulation within the cold region adjacent to the ice surface. The convective heat transfer between both regions seems to be limited mainly to the upper right corner. There the colliding cold and warm fluid layers intensify the heat transfer and effectively decrease the interface growth. The remaining centre and lower part of the interface is almost parallel to the cold wall, changing its shape very little with time. Here, the freezing process seems to be very little affected by the external convection. The cold water of temperature below 4°C remains trapped in the lower circulation region in the vicinity of the ice interface (Figure 2).

For air as an external fluid the effect of the heat flux through the top and bottom walls is small and the observed contact angle of the ice interface remains almost perpendicular. The best agreement with the numerical simulation was obtained assuming the adiabatic TBC for the side walls. To elucidate the effect of the side wall conduction, the additional experiments were performed (runs #5 - #9 in Table 1), with an external water bath surrounding the convection cavity. The general observation gained is that the flow pattern is less then expected sensible to the TBC at the side walls. Even such qualitative change of the external thermal conditions at the side walls hardly modified the freezing rate. Surprisingly, also relatively minor changes of ice interface were observed, mainly limited to the bottom region, where additional heat flux through the bottom wall curves slightly the ice interface (see Figure 4). This is not the case in our numerical modelling. The effect of wall conductivity on the ice interface is in the numerical results very strong. In comparison with the experiments, the heat flux through the side walls seems to be overestimated (Figure 3). At the moment we have no simple explanation of this discrepancy.

The temperature and velocity measurements confirm our general observation that the main fluid circulation takes place in the upper part of the cavity. The fluid temperature there is relatively uniform, close to that of the hot liquid. The average flow velocity is almost one order of magnitude higher then in the bottom region (Figure 5). Also the largest temperature gradients are present to the lower cavity region. With the progressing ice interface, the upper circulation diminishes in time on costs of the lower one. This slows down the ice growth, so then even after almost 4 hours of experimental observations less then one half of the cavity was frozen.

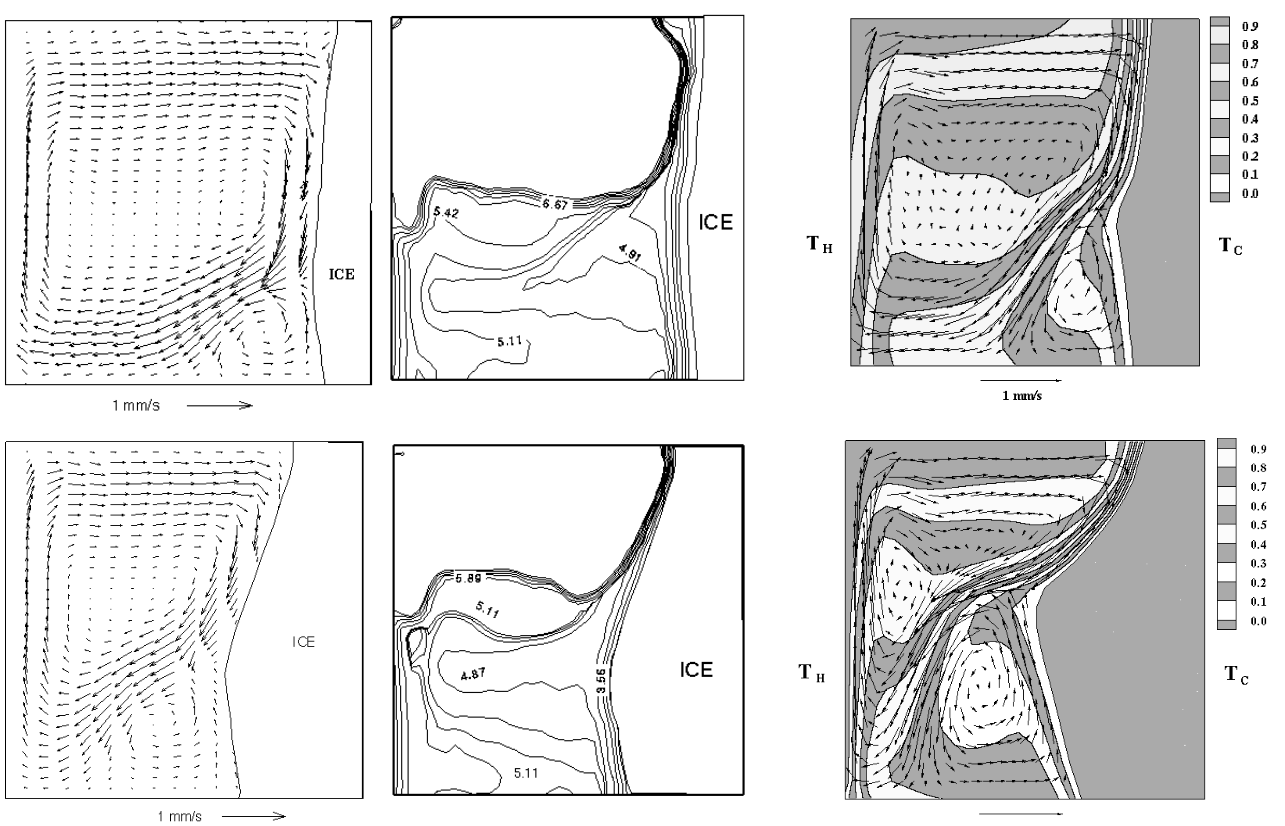

Figure 3. Measured velocity (left) and temperature (middle) fields; (right) - numerical results with 3-TBC; Time=500s (top) and 2600s (bottom); run #5: „warm start”, external water bath.

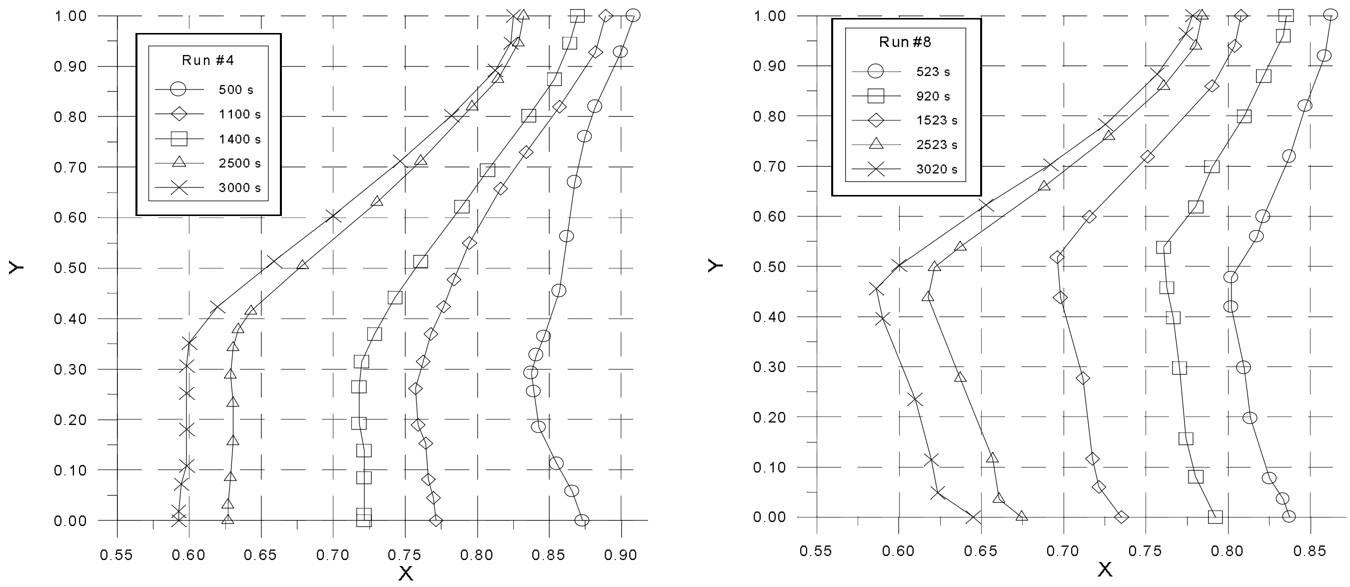

Figure 4. Interface profiles measured for run #4 (left) and run #8 (right) at selected time steps.

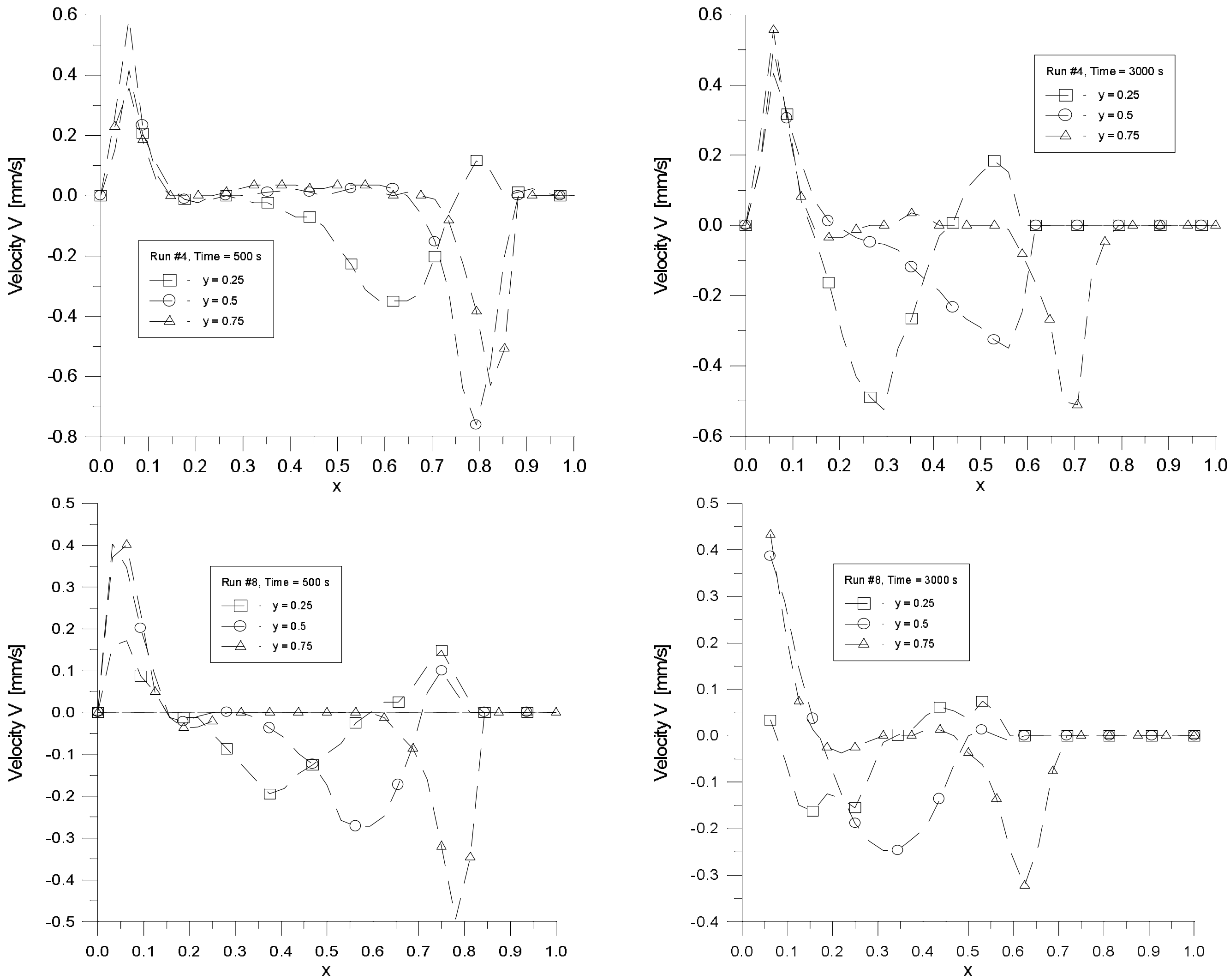

Figure 5. Vertical velocity component extracted from the PIV measurements. Velocity profiles along cavity taken at relative positions 0.25, 0.5 and 0.75 from the bottom. Upper row: run #4 (external air), lower row: run #8 (external water); time 500s (left) and 3000s (right) from the „warm start".

FINAL REMARKS

The experimental data on freezing of water in the cube-shaped cavity were collected in the purpose to create the reference for comparison with numerical results. The method of simultaneous measurements of the flow and temperature fields using liquid crystal tracers has been successfully applied to collect transient information on the flow. The collection of results illustrates complexity of the flow. Its interaction with the ice interface and with the side walls creates the two distinct flow regions. The upper adjacent to the hot wall with the strong clockwise convection of hot liquid, and the lower region adjacent to the interface with the slow counter-clockwise circulation of cold liquid. The mixing of both liquids seems to be small. Hence, the effect of natural convection on the freezing process is mostly limited to the interface intimate to the top of the cavity. The heat flux through the Plexiglas bottom and top walls has relatively small significance for the freezing process. The numerical simulation shows several differences in the front shape and flow pattern. It seems that the model used overestimates the wall conduction. Such effects observed in the experiment as supercooling, stagnant boundary layer at the bottom, imperfections of the ice structure, as well as non-ideal thermal contact between ice and the box surfaces, could be responsible for the discrepancies. Further numerical and experimental investigations seem to be necessary.

ACKNOWLEDGMENTS

The authors are indebted to G. Yeoh, G. de Vahl Davis and E. Leonardi (UNSW) for their computer code FREEZE3D. This work was supported by the Polish Scientific Committee (KBN Grant No. 3T09C00212). Part of the computations were performed on CRAY-CS6400 at the Warsaw University of Technology (COI).

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