A network model of self organization in geochemical flows

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Dissolution of rock fractures

$S^l, S^u$ – fracture surfaces

$h$ – aperture, $h/L \ll 1$
Fracture dissolution is a complex process...

- Fluid flow in a complicated geometry
- Reactant and product transport to and from fracture surfaces
- Chemical kinetics
- Geometry evolution
Experiment: KDP fracture
(Russell Detwiler et al., LLNL, 2003)

- sample size $15.2 \times 9.9$ cm
- initial mean aperture $\langle h_0 \rangle = 0.126$ mm
- dissolved until $\langle h \rangle = 2\langle h_0 \rangle$ at $Pe = 54$ and $Pe = 216$
- high resolution data on fracture topography
Aperture growth at $\text{Pe} = 216$ for $\langle h \rangle = 2 \langle h_0 \rangle$

- unsaturated fluid penetrates deep inside the fracture
- uniform dissolution
- lack of pronounced channels
Aperture growth at $\text{Pe} = 54$ for $\langle h \rangle = 2\langle h_0 \rangle$.

- channels form, grow, compete for the flow
- only few channels survive at the end
Channeling instability of the dissolution front

A small perturbation to the dissolution front is unstable.

The locally increased flow rate leads to increased dissolution, amplifying the perturbation (Ortoleva, 1987).

How do these perturbations develop in space and time?
Numerical study of dissolution in an idealized “fracture” geometry

- Pore-scale numerical simulations
- Simple initial fracture topography: plane channel with random obstacles
- No long-range correlations
- Transport-limited regime (large reaction rate)
Initial geometry

(fragment)
Initial flow field

\[ V(x, y) = \int_{s_i}^{s_u} \sqrt{v_x^2 + v_y^2} \, dz \] (total in-plane velocity flux)

(fragment)
Geometry evolution

$$Pe = \frac{V \langle h \rangle}{D}$$

$$Da = \frac{k}{V}$$
Flow maps

\[ Pe = \frac{V \langle h \rangle}{D} \]

\[ Da = \frac{k}{V} \]
Late-stage dissolution is dominated by channel competition

$$\langle \Delta h \rangle = 0.15 \langle h \rangle_0$$

$$\langle \Delta h \rangle = 0.5 \langle h \rangle_0$$

characteristic length between the channels is increasing
Scale invariance

\[ N(L) \propto L^{-\alpha}; \quad \alpha \approx 1.2 \]

\[ Pe = \frac{V\langle h \rangle}{D} = 32 \]

The cumulative distribution of channel lengths
Channel competition: flow capturing
Channel competition: flow capturing

Upstream: towards long channel
Downstream: away from long channel

Pressure gradient in long channel is steeper (higher flow rate)
Pressure at long channel is lower than the short channel near the inlet
But higher near the outlet
The undissolved medium has high resistivity ($\rho_U$)

Channels have low resistivity ($\rho_C$)

Resistance proportional to length

$$R_1 = R_4 = \rho_U L_A, \quad R_5 = \rho_M (L_B - L_A) \ldots$$
Evolution of the resistor network

\[
\frac{dL_B}{dt} = I_2 \\
\frac{dL_A}{dt} = I_4
\]

n|e|d|d|le|l|k||i|l|e|n|l||i|c|h|a|l|s| growing only at the top
Multi-channel model ~1200 channels

Network model shows the same scaling properties as the dissolving fracture system

\[ N(L) \sim L^{-\alpha} \]
\[ \alpha = 1.25 \]
Viscous fingering

For the front becomes unstable

Hele-Shaw cell

For $\mu_B > \mu_A$ the front becomes unstable
Experiment

radial Hele-Shaw cell

Camera

silicon oil ≈ 0.1 mm

air

oil
Front instability

Swinney et al. (2002)
Late stages

Swinney et. al (2002)

Hertzberg, Sweetman (2005)
Experiment needed...

Viscous fingering in a network of channels

$\log L$

$\log N$

scale invariance?

$\alpha = ?$

$N(L) \sim L^{-\alpha}$
Conclusions

- The dissolution patterns in a porous medium show scale-invariant properties
- Core of the interaction between the channels is capture of the flow form the shorter by the longer ones
- A model of interaction between dissolving channels was constructed by mapping the system into an evolving resistor network
- Network model shows the same nontrivial scaling features as the dissolving fracture system
- A similar scaling should be observed in the viscous fingering phenomena in the network of channels