Slow viscous migration of solid particles in a conducting liquid under electric and magnetic fields

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Outline

1) Phenomenon and motivations

2) Adopted assumptions

3) Available results. Case of a spherical particle

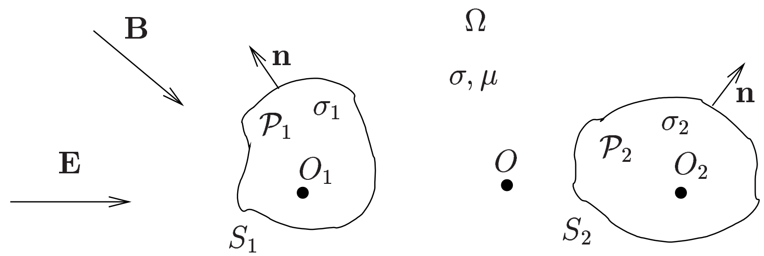
4) Asymptotic analysis for two distant spheres

5) Boundary formulation for arbitrary $N-$particule clusters

6) Advocated numerical strategy and numerical resultats

7) Conclusions

$N \geq 1$ solid particles $\mathcal{P}_n$



Action of $\mathbf{E}$ and $\mathbf{B}$ (Leenov & Kolin 1954, Marty & Alemany 1984)?

- Liquid: current $\mathbf{j} = \sigma(\mathbf{E} - \nabla\phi + \mathbf{u} \wedge \mathbf{B})$, Lorentz body force $\mathbf{f} = \mathbf{j} \wedge \mathbf{B}$
- $\mathcal{P}_n$: current $\mathbf{j}_n = \sigma_n(\mathbf{E} - \nabla\phi_n + \mathbf{u}^{(n)} \wedge \mathbf{B})$, Lorentz body force $\mathbf{f}_n = \mathbf{j}_n \wedge \mathbf{B}$

Migrations triggered by: $\mathbf{E}, \mathbf{B}$ and $\sigma - \sigma_n$!

Unknown quantities: liquid flow and rigid-body motions $\mathbf{u}^{(n)}$

$$(\mathbf{u}, P) \text{ and } (\mathbf{U}^{(n)}, \mathbf{\Omega}^{(n)})?$$

Potential applications?

- Solid impurities removal in conducting liquids (liquid metals, liquid glass)
- Particules separation or/and deposition on solid boundaries

Previous results

- Kolin 1953, Leenov & Kolin 1954. Analytical solution for a conducting sphere
- Marty & Alemany 1984: experiments for cylindrical and spherical bodies
- Moffatt & Sellier 2002: symmetry considerations for an insulating particule
- Sellier 2003, 2004, 2005, 2007: boundary formulation for arbitrarily-shaped insulating particules, analytical solution for conducting ellipsoids, numerical solution for conducting and arbitrarily-shaped particules, particule-particule interactions for two insulating particules.

Some basic issues

- Case of arbitrary collections of solid and conducting particules: asymptotic and numerical analysis. Present work!
- Case of bubbles: under investigation.
- Case of wall-particle interactions. Sellier 2006: semi-analytical solution for a sphere. To be further extended to several particules.
- Case of droplet? Micro-mixing inside the droplets?

Adopted assumptions

• Small particules with typical length and velocity scales a and U such

$$Re = \rho U a/\mu \ll 1, \quad R_m = \mu_m \sigma U a \ll Re, \quad M^2 = \sigma B^2 a^2 \ll 1$$

Consequences: decoupled electrostatic and flow problems

$$\mathbf{B}, \quad \mathbf{j} \sim \sigma(\mathbf{E} - \nabla\phi), \quad \mathbf{j}_n \sim \sigma_n(\mathbf{E} - \nabla\phi_n)$$

• Electrostatic problem

$$\nabla^2\phi_n = 0 \text{ in } \mathcal{P}_n, \; \nabla^2\phi = 0 \text{ in } \Omega, \quad \nabla\phi \to \mathbf{0} \text{ if } r \to \infty,$$

$$\sigma_n(\mathbf{E} - \nabla\phi_n).\mathbf{n} = \sigma(\mathbf{E} - \nabla\phi).\mathbf{n} \text{ and } \phi = \phi_n \text{ on } S_n$$

Well-posed. Permits to calculate the net force and torque exerted on $\mathcal{P}_n$:

$$\mathbf{F}'_n = \sigma_n[\int_{\mathcal{P}_n}(\mathbf{E} - \nabla\phi_n)dv] \wedge \mathbf{B}, \; \mathbf{G}'_n = \sigma_n \int_{\mathcal{P}_n} \mathbf{O_n M} \wedge [(\mathbf{E} - \nabla\phi_n) \wedge \mathbf{B}]dv$$

Henceforth, we use the decompositions and notations

$$\mathbf{F}'_n = \sigma_n[\mathcal{V}_n(\mathbf{E} \wedge \mathbf{B}) - \mathbf{A}_n \wedge \mathbf{B}], \quad \mathbf{G}'_n = \sigma_n[\mathbf{C}_n \wedge (\mathbf{E} \wedge \mathbf{B}) - \mathbf{B}_n]$$

$$\mathbf{A}_n = \int_{\mathcal{P}_n} \nabla\phi_n dv, \; \mathbf{C}_n = \int_{\mathcal{P}_n} \mathbf{O_n M} dv, \; \mathbf{B}_n = \int_{\mathcal{P}_n} \mathbf{O_n M} \wedge (\nabla\phi_n \wedge \mathbf{B})dv$$

- Flow problem for $(\mathbf{u}, p + \sigma(\mathbf{E} \wedge \mathbf{B}).\mathbf{x})$

$$\nabla.\mathbf{u} = 0, \quad \mu\nabla^2\mathbf{u} = \nabla p + \sigma\nabla\phi \wedge \mathbf{B} \ \text{ in } \Omega$$

$$(\mathbf{u}, p) \to (\mathbf{0}, 0) \text{ if } r \to \infty,$$

$$\mathbf{u} = \mathbf{U}^{(n)} + \boldsymbol{\Omega}^{(n)} \wedge \mathbf{O_n M} \ \text{ on } S_n$$

- Non-uniform body force acting in the entire liquid domain!

- If $(\mathbf{u}, p)$ has stress tensor $\boldsymbol{\sigma}$

the flow exerts on $\mathcal{P}_n$ the net force and torque:

$$\mathbf{F}_n = \int_{S_n} \boldsymbol{\sigma}.\mathbf{n} dS - \sigma\mathcal{V}_n(\mathbf{E} \wedge \mathbf{B})$$

$$\mathbf{G}_n = \int_{S_n} \mathbf{O_n M} \wedge [\boldsymbol{\sigma}.\mathbf{n}] dS - \sigma[\int_{\mathcal{P}_n} \mathbf{O_n M} dv] \wedge (\mathbf{E} \wedge \mathbf{B})$$

- Additional relations $\mathbf{U}^{(n)}, \boldsymbol{\Omega}^{(n)}$?

Particules of negligible inertia

$$\mathbf{F}_n + \mathbf{F}'_n = \mathbf{0}, \ \mathbf{G}_n + \mathbf{G}'_n = \mathbf{0}$$

- Quite a very few analytical solutions (sphere, ellipsoids)
- Numerical method? Iterative procedure? High cpu-time cost and poor accuracy!

Analytical solution for a conducting sphere

•Sphere with radius a_n and conductivity σ_n (Leenov & Kolin 1954)

$$\mathbf{\Omega}_0^{(n)} = \mathbf{0}, \ \mathbf{U}_0^{(n)} = \frac{\sigma a_n^2 C_n}{3\mu}(\mathbf{E} \wedge \mathbf{B}), \ C_n = \frac{\sigma_n - \sigma}{\sigma_n + 2\sigma}$$

•Possible to calculate the velocity field $\mathbf{u}$ about the sphere

It the sphere has center O_n and $\mathbf{x}_n = \mathbf{O}_n, r_n = |\mathbf{x}_n|$ then

$$\mathbf{u} = \frac{\sigma a_n^3 C_n}{4\mu r_n}[(\frac{a}{r_n})^2 - 1][(\mathbf{E}.\mathbf{x}_n)\mathbf{B} + (\mathbf{B}.\mathbf{x}_n)\mathbf{E}] \wedge \frac{\mathbf{x}_n}{r_n^2}$$
$$+\frac{3a_n}{4r_N}[\mathbf{U}_0^{(n)} + \frac{(\mathbf{U}_0^{(n)}.\mathbf{x}_n)\mathbf{x}_n}{r_n^2}] + \frac{a_n^3}{4r_n^3}[\mathbf{U}_0^{(n)} - \frac{3(\mathbf{U}_0^{(n)}.\mathbf{x}_n)\mathbf{x}_n}{r_n^2}] + a_n^3\mathbf{\Omega}^{(1)} \wedge \frac{\mathbf{x}_n}{r_n^3}$$

•Fruitful resultats for several distant spheres

- Using the so-called reflection method

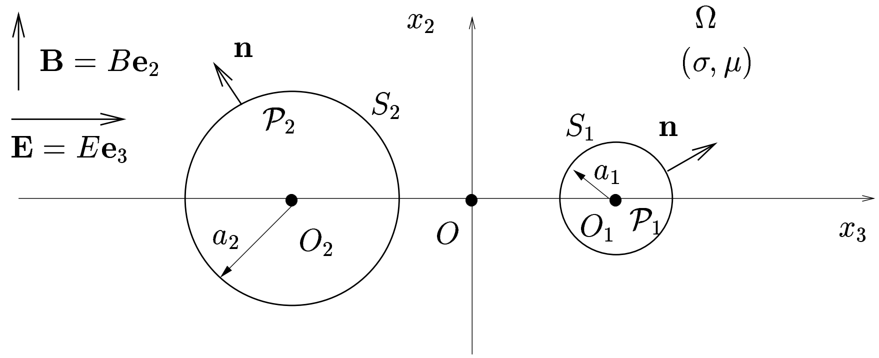
- Case of 2 distant spheres (today)

- Case of several “equally” distant spheres: achieved (too long to present)

- Spheres interactions: short or long range ones?

- Sensitivity to $(\mathbf{E}, \mathbf{B})$?

Case of 2 distant spheres



- Here $d = O_1O_2 \gg a_1 + a_2$ and $\mathbf{e}_{21} := \mathbf{O_2O_1}/d$.

The asymptotic analysis yields

$$\mathbf{U}^{(1)} \sim \mathbf{U}_0^{(1)} + \frac{3}{4}\Big(\frac{a_2}{d}\Big)\Big\{\mathbf{U}_0^{(2)} + (\mathbf{U}_0^{(2)}.\mathbf{e}_{21})\mathbf{e}_{21} - \frac{\sigma a_2^2 C_2}{3\mu}\mathbf{V}\Big\} +$$

$$\Big(\frac{a_2}{d}\Big)^3\Big\{\frac{\sigma a_2^2 C_2}{4\mu}\mathbf{V} + \frac{\sigma a_1^2 C_2}{6\mu}(1 - 2C_1)\mathbf{E}' \wedge \mathbf{B} + \frac{a_2^2 + 2a_1^2}{4a_2^2}[\mathbf{U}_0^{(2)} - 3(\mathbf{U}_0^{(2)}.\mathbf{e}_{21})\mathbf{e}_{21}]\Big\},$$

$$\mathbf{\Omega}^{(1)} \sim \frac{3}{4}\Big(\frac{a_2}{d}\Big)^2\Big\{\frac{\mathbf{U}_0^{(2)} \wedge \mathbf{e}_{21}}{a_2} + \frac{\sigma a_2 C_2}{3\mu}[(\mathbf{E}.\mathbf{B}) - 3(\mathbf{E}.\mathbf{e}_{21})(\mathbf{B}.\mathbf{e}_{21})]\mathbf{e}_{21}\Big\},$$

$$\mathbf{E}' = \mathbf{E} - 3(\mathbf{E}.\mathbf{e}_{21})\mathbf{e}_{21}, \;\; \mathbf{V} = [(\mathbf{E}.\mathbf{e}_{21})\mathbf{B} + (\mathbf{B}.\mathbf{e}_{21})\mathbf{E}] \wedge \mathbf{e}_{21}$$

By superposition sufficient to deal with 5 different Cases

•Case (i): $\mathbf{B}.\mathbf{e}_{21} = 0$ and $\mathbf{E} \wedge \mathbf{e}_{21} = \mathbf{0}$

$$\mathbf{U}^{(1)} \sim \mathbf{U}_0^{(1)} + \{\frac{3}{2}(\frac{a_2}{d}) + (\frac{a_2}{d})^3[\frac{(8C_1 - 1)a_1^2 - a_2^2}{4a_2^2}]\}\mathbf{U}_0^{(2)},$$

$$\mathbf{\Omega}^{(1)} \sim \frac{3}{4}(\frac{a_2}{d})^2 \frac{\mathbf{U}_0^{(2)} \wedge \mathbf{e}_{21}}{a_2}$$

•Case (ii): $\mathbf{E}.\mathbf{e}_{21} = 0$ and $\mathbf{B} \wedge \mathbf{e}_{21} = \mathbf{0}$. $\mathbf{\Omega}^{(1)}$ still given as above and

$$\mathbf{U}^{(1)} \sim \mathbf{U}_0^{(1)} + (\frac{a_2}{d})^3[1 + (1 - C_1)\frac{a_1^2}{a_2^2}]\mathbf{U}_0^{(2)}$$

•Case (iii): $\mathbf{E}.\mathbf{B} = \mathbf{E}.\mathbf{e}_{21} = \mathbf{B}.\mathbf{e}_{21} = 0$. This time $\mathbf{\Omega}^{(1)} = \mathbf{0}$ and

$$\mathbf{U}^{(1)} \sim \mathbf{U}_0^{(1)} + \{\frac{3}{2}(\frac{a_2}{d}) - (\frac{a_2}{d})^3[\frac{a_2^2 + 2a_1^2}{2a_2^2}]\}\mathbf{U}_0^{(2)}$$

•Cases (iv)-(v) with $\mathbf{E} \wedge \mathbf{B} = \mathbf{0}$.
$\mathbf{E}.\mathbf{e}_{21} = 0$ for (iv) and $\mathbf{E} \wedge \mathbf{e}_{21} = 0$ for (v). Here $U^{(1)} = \mathbf{0}$ and

$$\mathbf{\Omega}^{(1)} \sim \frac{3}{4}(\frac{a_2}{d})^2 \frac{\sigma a_2 C_2}{3\mu}(\mathbf{E}.\mathbf{B})\mathbf{e}_{21}\ (iv),\quad \mathbf{\Omega}^{(1)} \sim -\frac{3}{2}(\frac{a_2}{d})^2 \frac{\sigma a_2 C_2}{3\mu}(\mathbf{E}.\mathbf{B})\mathbf{e}_{21}\ (v)$$

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Surface quantities for the electrostatic problem

- Electrostatic problem

$$\nabla^2\phi_n = 0 \text{ in } \mathcal{P}_n, \quad \nabla^2\phi = 0 \text{ in } \Omega, \quad \nabla\phi \to \mathbf{0} \text{ as } r \to \infty,$$
$$\sigma_n(\mathbf{E} - \nabla\phi_n).\mathbf{n} = \sigma(\mathbf{E} - \nabla\phi).\mathbf{n} \text{ and } \phi = \phi_n \text{ on } S_n$$

- Polarisation charge density q on $S = S_1 \cup ... \cup S_N$

$$4\pi\psi(M) = \int_S q(P)dS/PM \text{ in } I\!R^3, \;\; \phi = \psi \;\text{ in }\; \Omega, \;\; \phi_n = \psi \;\text{ in }\; \mathcal{P}_n$$

- Inside $\mathcal{P}_n$

$$\mathbf{F}'_n = \sigma_n[\mathcal{V}_n(\mathbf{E} \wedge \mathbf{B}) - \mathbf{A}_n \wedge \mathbf{B}], \quad \mathbf{G}'_n = \sigma_n[\mathbf{C}_n \wedge (\mathbf{E} \wedge \mathbf{B}) - \mathbf{B}_n]$$

$$\mathbf{A}_n = \int_{\mathcal{P}_n} \nabla\phi_n dv = \int_{S_n} \phi\mathbf{n}dS$$

$$\mathbf{C}_n = \int_{\mathcal{P}_n} \mathbf{O_n M}dv, \qquad h(M) = \frac{1}{4\pi}\int_S \frac{q(P)\mathbf{MP}.\mathbf{n}(P)}{PM}dS$$

$$\mathbf{B}_n = \int_{\mathcal{P}_n} \mathbf{O_n M} \wedge (\nabla\phi_n \wedge \mathbf{B})dv = \int_{S_n} \{\phi[\mathbf{B}.\mathbf{O_n M}]\mathbf{n} - (\mathbf{O_n M}.\mathbf{n})\mathbf{B}] + h\mathbf{B}\}dS$$

- One solely requires q and $\phi = \phi_n$ on S

Surface quantities for the flow problem

- Flow problem $(\mathbf{u}, p + \sigma(\mathbf{E} \wedge \mathbf{B}).\mathbf{x})$

$$\nabla.\mathbf{u} = 0, \quad \mu\nabla^2\mathbf{u} = \nabla p + \sigma\nabla\phi \wedge \mathbf{B} \quad \text{in } \Omega$$

$$(\mathbf{u}, p) \to (\mathbf{0}, 0) \text{ if } r \to \infty,$$

$$\mathbf{u} = \mathbf{U}^{(n)} + \mathbf{\Omega}^{(n)} \wedge \mathbf{O_n M} \quad \text{on } S_n$$

- $6N$ Stokes flows $(\mathbf{u}_T^{(n),i}, p_T^{(n),i}), (\mathbf{u}_R^{(n),i}, p_R^{(n),i})$

$$\mathbf{f}' = \mathbf{0}, \quad \mathbf{u}_T^{(n),i} = \delta_{nm}\mathbf{e}_i, \quad \mathbf{u}_R^{(n),i} = \delta_{nm}\mathbf{e}_i \wedge \mathbf{O_n M} \quad \text{on } S_m$$

Associated surface tractions $\mathbf{f}_L^{(n),i}$ on S and coefficients

$$-\mu A_{(m),L}^{(n),i,j} = \int_{S_m} \mathbf{e}_j.\mathbf{f}_L^{(n),i} dS_m, \; -\mu B_{(m),L}^{(n),i,j} = \int_{S_m} (\mathbf{e}_j \wedge \mathbf{O_m M}).\mathbf{f}_L^{(n),i} dS_m$$

- Reciprocal identity

$$\int_S [\mathbf{u}.\boldsymbol{\sigma}'.\mathbf{n} - \mathbf{u}'.\boldsymbol{\sigma}.\mathbf{n}] dS = \int_\Omega [\mathbf{u}'.\mathbf{f} - \mathbf{u}.\mathbf{f}'] d\Omega$$

- Volume integrals

$$\mathcal{L}[\mathbf{f}_L^{(n),i}] = 8\pi\mu \int_\Omega \mathbf{u}_L^{(n),i}.[\nabla\phi \wedge \mathbf{B}] d\Omega$$

Linear system

- If $\delta_n = \sigma_n/\sigma$, $\mathbf{U}^{(n)} = U_j^{(n)}\mathbf{e}_j$ and $\boldsymbol{\Omega}^{(n)} = \Omega_j^{(n)}\mathbf{e}_j$

$$A_{(m),T}^{(n),i,j}U_j^{(m)} + B_{(m),T}^{(n),i,j}\omega_j^{(m)} = \frac{\sigma}{\mu}\{(\delta_n - 1)\mathcal{V}_n[\mathbf{E} \wedge \mathbf{B}] - \delta_n \mathbf{A}_n \wedge \mathbf{B} + \frac{\mathcal{L}[\mathbf{f}_T^{(n),i}]}{8\pi}\}.\mathbf{e}_i$$

$$A_{(m),R}^{(n),i,j}U_j^{(m)} + B_{(m),R}^{(n),i,j}\omega_j^{(m)} = \frac{\sigma}{\mu}\{(\delta_n - 1)\mathbf{C}_n \wedge [\mathbf{E} \wedge \mathbf{B}] - \delta_n \mathbf{B}_n + \frac{\mathcal{L}[\mathbf{f}_R^{(n),i}]}{8\pi}\}.\mathbf{e}_i$$

- With (omitted details!)

$$\begin{aligned}
\mathcal{L}[\mathbf{v}] \;=\; & \int_S \int_S [\mathbf{v}(P).\frac{\mathbf{PM}}{PM}][\nabla\phi(M) \wedge \mathbf{B}].\mathbf{n}(M)dS_P dS_M \\
& - \int_S \int_S \mathbf{v}(P).[\nabla\phi(M) \wedge \mathbf{B}]\frac{\mathbf{PM}.\mathbf{n}(M)}{PM}dS_P dS_M \\
& + \int_S \int_S \epsilon_{kmn} PM[\mathbf{v}.\mathbf{e}_k](P)[\mathbf{B}.\mathbf{e}_n][\nabla(\phi_{,m}).\mathbf{n}](M)dS_P dS_M
\end{aligned}$$

- In summary, one solely needs to calculate the surface quantities

$$q, \quad \phi, \quad \mathbf{f}_L^{(n),i} = \sigma_L^{(n),i}.\mathbf{n}, \quad \phi_{,m} = \frac{\partial\phi}{\partial x_m}, \quad \nabla(\phi_{,m}).\mathbf{n}$$

Relevant boundary-integral equations

- One Fredholm boundary-integral equations of the second kind

$$2\pi[\frac{1+\delta_n}{1-\delta_n}]q(M) + \int_S q(P)\frac{\mathbf{PM}.\mathbf{n}(M)dS}{PM^3} = -4\pi[\mathbf{E}.\mathbf{n}](M), M \text{ on } S_n$$

$6N$ Fredholm boundary-integral equations of the first kind

$$[\mathbf{u}_L^{(n),i}.\mathbf{e}_k](M) = -\int_S \{\frac{\delta_{jk}}{PM} + \frac{(\mathbf{PM}.\mathbf{e}_j)(\mathbf{PM}.\mathbf{e}_k)}{PM^3}\}[\frac{\mathbf{f}_L^{(n),i}.\mathbf{e}_j}{8\pi\mu}](P)dS$$

For ψ harmonic in Ω

$$-4\pi\psi(M) + \int_S [\psi(P) - \psi(M)]\frac{\mathbf{PM}.\mathbf{n}(P)}{PM^3}dS = \int_S \frac{[\nabla\psi.\mathbf{n}](P)}{PM}dS$$

- For $\psi = \phi$ this gives ϕ on S from $\nabla\phi.\mathbf{n} = \mathbf{E}.\mathbf{n} + \delta_n q/(1-\delta_n)$ on S_n
- Provides $\phi_{,m}$ on S and $\nabla\phi.\mathbf{n}$
- For $\psi = \phi_{,m}$ one gets $\nabla(\phi_{,m}).\mathbf{n}$ on S

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Numerical method and resultats

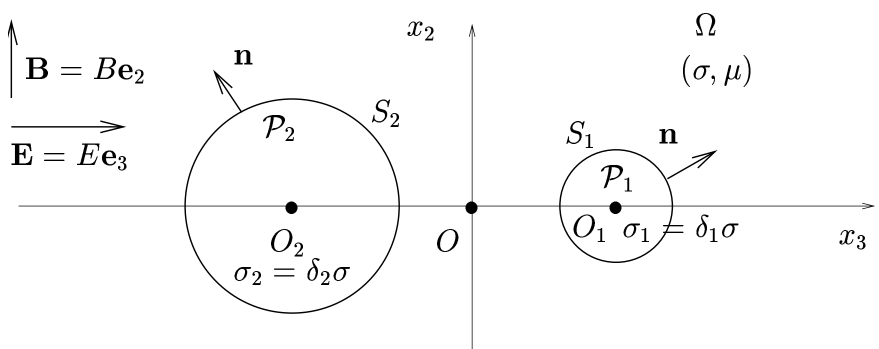
- P2 6-node triangular boundary elements
- Gaussian elimination (dense and non symmetric influence matrix)

- Numerical comparisons for a sphere with radius a

$$\mathbf{U} = \sigma a^2 c(\delta)(\mathbf{E} \wedge \mathbf{B})/\mu$$

M	$\delta = 0$	$\delta = 0.5$	$\delta = 2$	$\delta = 5$
74	-0.17447	-0.07025	0.08968	0.21399
242	-0.16756	-0.06704	0.08383	0.19169
1058	-0.16677	-0.06669	0.08335	0.19048
exact	-0.16667	-0.06667	0.08333	0.19048

- Case of 2 spheres



Cas (i): $(E\mathbf{e}_3, B\mathbf{e}_2)$, Cas (ii): $(E\mathbf{e}_2, B\mathbf{e}_3)$, Cas (iii): $(E\mathbf{e}_1, B\mathbf{e}_2)$

$$u_j^{(n)}(\lambda) = \frac{\mu U_j^{(n)}}{\sigma a_1^2 |\mathbf{E}||\mathbf{B}|}, \quad w_j^{(n)}(\lambda) = \frac{\mu \Omega_j^{(n)}}{\sigma a_1 |\mathbf{E}||\mathbf{B}|}, \quad 0 \leq \lambda = \frac{a_1 + a_2}{O_1 O_2} < 1$$

Adopted meshes for 2 close spheres

$\lambda = 0.9, a_2 = 2a_1$ and M nodal points on each S_n

Insulating spheres: $\delta_1 = \delta_2 = 0$

M	74	242	530	1058
$u_1^{(1)}$	-0.34687	-0.36356	-0.36519	-0.36560
$w_2^{(1)}$	0.13461	0.13596	0.13574	0.13574
$u_1^{(2)}$	-0.68519	-0.69862	-0.69927	-0.69962
$w_2^{(2)}$	-0.01657	-0.01652	-0.01634	-0.01633

Conducting spheres: $\delta_1 = 2$ and $\delta_2 = 4$

M	74	242	530	1058
$u_1^{(1)}$	0.26310	0.25623	0.25508	0.25505
$w_2^{(1)}$	-0.12968	-0.12598	-0.12568	-0.12557
$u_1^{(2)}$	0.70147	0.67336	0.67042	0.66994
$w_2^{(2)}$	0.00326	0.00347	0.00339	0.00339

By superposition sufficient to deal with 5 different Cases

•Case (i): $\mathbf{B}.\mathbf{e}_{21} = 0$ and $\mathbf{E} \wedge \mathbf{e}_{21} = \mathbf{0}$

$$\mathbf{U}^{(1)} \sim \mathbf{U}_0^{(1)} + \{\frac{3}{2}(\frac{a_2}{d}) + (\frac{a_2}{d})^3[\frac{(8C_1 - 1)a_1^2 - a_2^2}{4a_2^2}]\}\mathbf{U}_0^{(2)},$$

$$\mathbf{\Omega}^{(1)} \sim \frac{3}{4}(\frac{a_2}{d})^2 \frac{\mathbf{U}_0^{(2)} \wedge \mathbf{e}_{21}}{a_2}$$

•Case (ii): $\mathbf{E}.\mathbf{e}_{21} = 0$ and $\mathbf{B} \wedge \mathbf{e}_{21} = \mathbf{0}$. $\mathbf{\Omega}^{(1)}$ still given as above and

$$\mathbf{U}^{(1)} \sim \mathbf{U}_0^{(1)} + (\frac{a_2}{d})^3[1 + (1 - C_1)\frac{a_1^2}{a_2^2}]\mathbf{U}_0^{(2)}$$

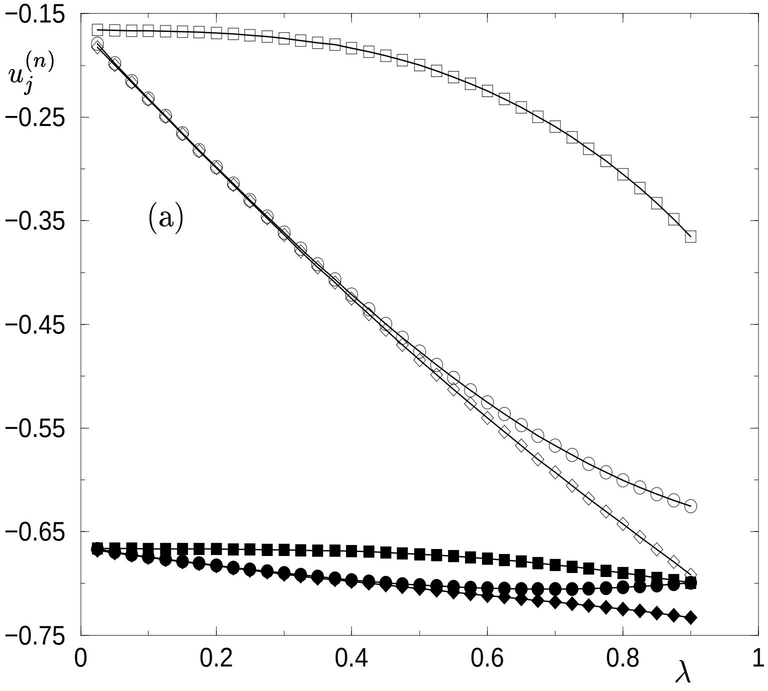
•Case (iii): $\mathbf{E}.\mathbf{B} = \mathbf{E}.\mathbf{e}_{21} = \mathbf{B}.\mathbf{e}_{21} = 0$. This time $\mathbf{\Omega}^{(1)} = \mathbf{0}$ and

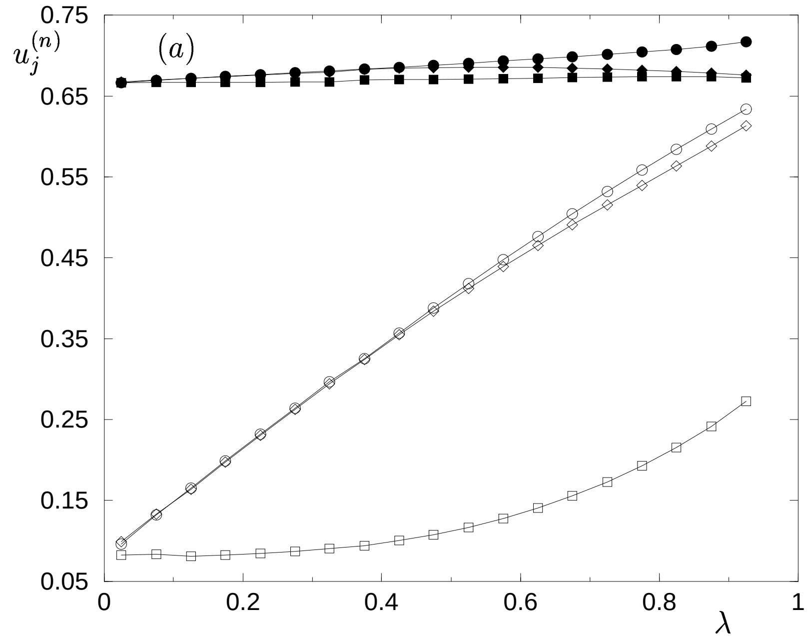
$$\mathbf{U}^{(1)} \sim \mathbf{U}_0^{(1)} + \{\frac{3}{2}(\frac{a_2}{d}) - (\frac{a_2}{d})^3[\frac{a_2^2 + 2a_1^2}{2a_2^2}]\}\mathbf{U}_0^{(2)}$$

•Cases (iv)-(v) with $\mathbf{E} \wedge \mathbf{B} = \mathbf{0}$.
$\mathbf{E}.\mathbf{e}_{21} = 0$ for (iv) and $\mathbf{E} \wedge \mathbf{e}_{21} = 0$ for (v). Here $U^{(1)} = \mathbf{0}$ and

$$\mathbf{\Omega}^{(1)} \sim \frac{3}{4}(\frac{a_2}{d})^2 \frac{\sigma a_2 C_2}{3\mu}(\mathbf{E}.\mathbf{B})\mathbf{e}_{21} \ (iv), \quad \mathbf{\Omega}^{(1)} \sim -\frac{3}{2}(\frac{a_2}{d})^2 \frac{\sigma a_2 C_2}{3\mu}(\mathbf{E}.\mathbf{B})\mathbf{e}_{21} \ (v)$$

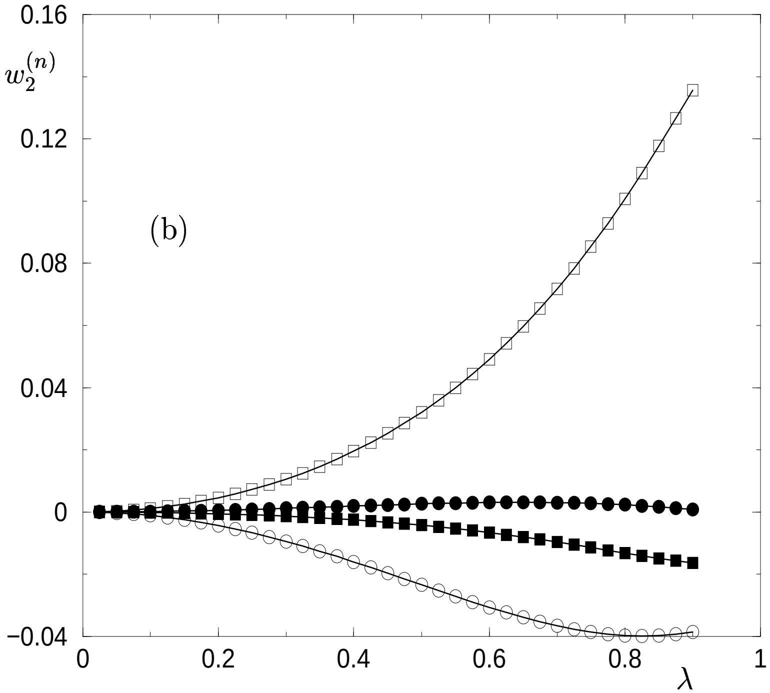
Translational velocities for 2 spheres $(a_2 = 2a_1)$

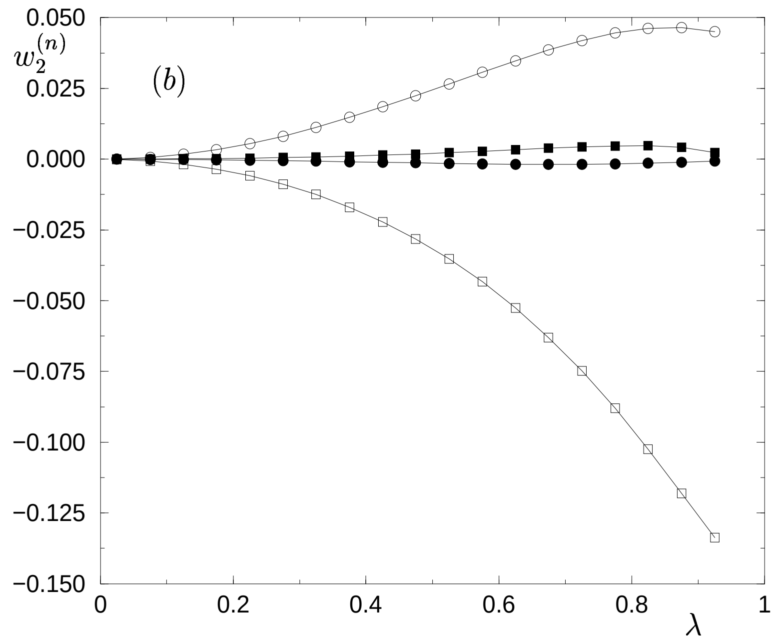




Functions $-u_1^{(1)}(\circ)$ and $-u_1^{(2)}(\bullet)$ Cas(i),
$u_1^{(1)}(\square)$ and $u_1^{(2)}(\blacksquare)$ Cas (ii),
$u_3^{(1)}(\diamond)$ and $u_3^{(2)}(\blacklozenge)$ Cas (iii)

Angular velocities for 2 spheres $(a_2 = 2a_1)$



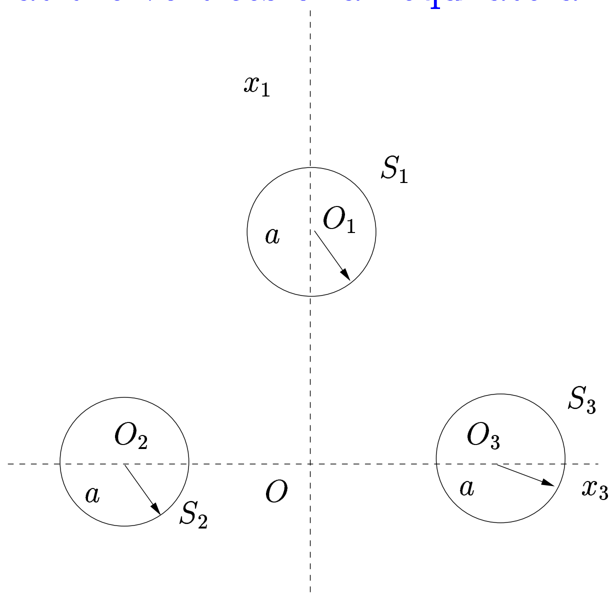


Functions $w_2^{(1)}(\circ)$ and $w_2^{(2)}(\bullet)$ Cas (i),
$w_2^{(1)}(\square)$ and $w_2^{(2)}(\blacksquare)$ Cas (ii)

Example of strong sphere-sphere interactions

3 spheres with radius a

located at the vortices of an equilateral triangle



$$O_1O_2 = O_1O_3 = O_2O_3 = d > 2a, \quad \lambda = 2a/d$$

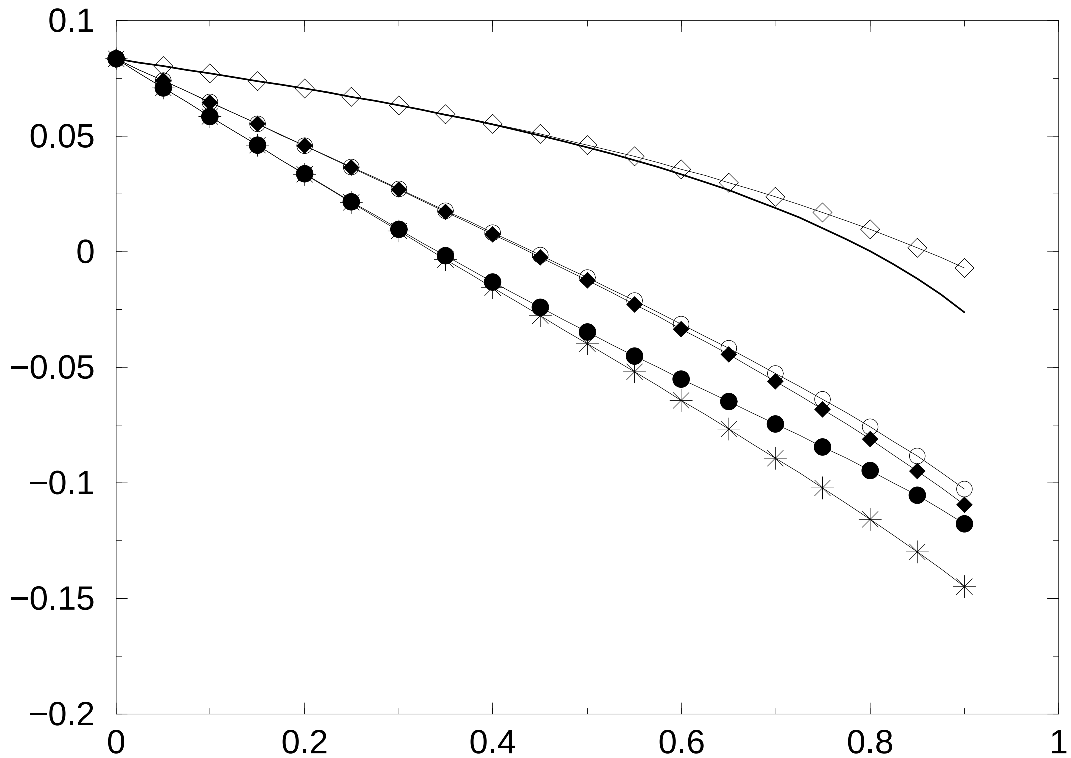
$\sigma_2 = \sigma_3 = 0$ and $\sigma_1 = 2$

$\mathbf{U}^{(1)}$ has only one non-zero Cartesian component $\sigma a^2 |\mathbf{E}||\mathbf{B}| u_i/\mu$

if $\mathbf{E}$ and $\mathbf{B}$ are aligned wiht unit vectors $\mathbf{e}_k$

$u_i(\lambda)$?

Non-zero components $u_i(\lambda)$ versus λ



Functions $u_1(\circ)$ for $\mathbf{E} = e_2$ and $\mathbf{B} = e_3$,
$-u_1(\bullet)$ for $\mathbf{E} = e_3$ and $\mathbf{B} = e_2$,
$-u_2(\blacklozenge)$ for $\mathbf{E} = e_1$ and $\mathbf{B} = e_3$, $u_2(\lozenge)$ for $\mathbf{E} = e_3$ and $\mathbf{B} = e_1$,
$u_3(*)$ for $\mathbf{E} = e_1$ and $\mathbf{B} = e_2$, $-u_3(—)$ for $\mathbf{E} = e_2$ and $\mathbf{B} = e_1$

Conclusions

• Useless to determine the liquid flow and
disturbed electric field in the entire unbounded fluid domain Ω!

• Efficient boundary approach for arbitrary $N-$particule clusters

• The BEM is suitable: good accuracy at a resonable cpu time cost!
(Putting 242 nodal points on each sphere is quite sufficient even for rather close spheres)

• Particle-particle interactions may be either strong or weak and
deeply depend upon $\mathbf{E}, \mathbf{B}$ and the particle nature (shape, location, conductivity)

Future investigations

• Bubbles and droplets!

• Solid boundaries: competition between wall-particle
and particle-particle interactions